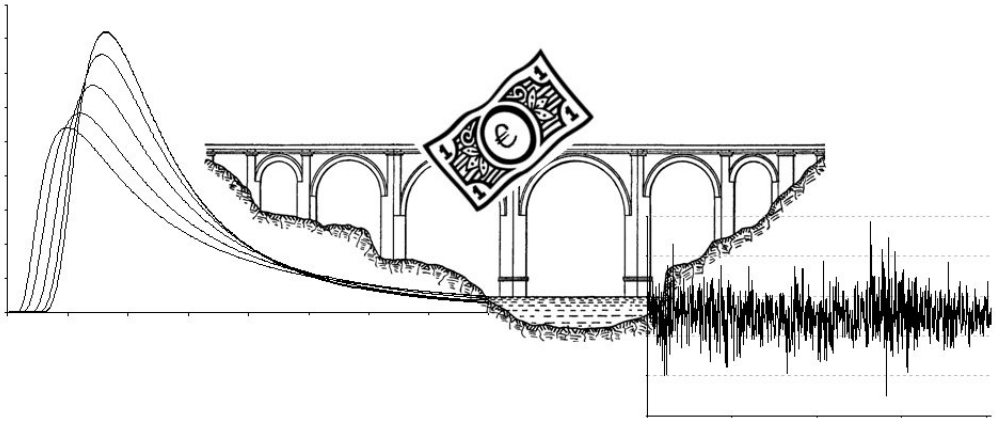
# Risk & Extreme Values in Insurance and Finance

## Extended Abstracts



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[1]Edition partially supported by FCT/OE and PTDC/FEDER.

# Preface

This is a two-day workshop aimed to build a bridge between researchers and practitioners. The focus is on *Financial Extreme Values* and *Extreme Value Theory* (EVT) as a practical risk mitigation tool. The *Workshop on Risk & Extreme Values in Insurance and Finance* is held in the framework of the 100-year anniversary of the Faculty of Sciences of the University of Lisbon (FCUL), in 2011. The conference represents a unique event which will bring together in Portugal the three authors of the book "*Modelling Extremal Events for Insurance and Finance*" — Paul Embrechts (Zurich, Switzerland), Claudia Klüppelberg (München, Germany), and Thomas Mikosch (Copenhagen, Denmark) — and presents new challenges for the second decade of the 21st century, both for academics and practitioners of the financial world. The "EKM book" constitutes an encyclopedic handbook of theory and statistical praxis and has been classified as of

> "great value to actuaries and statisticians in the fields concerned and at the same time a useful and well motivated text book for those who need a guide for entering the area without getting lost either in pure theory or messy practice" — *Mathematics Today*.

More than 10 years after the first edition in 1997, there is consensus all over the world that it represents

> "...the indispensable starting point for anyone interested in contemporary applications and extensions of classical EVT." — *Extremes*.

Three other scientists specialized in the interplay between probability, statistics, risk and finance will give invited addresses: Holger Drees (Hamburg, Germany) Casper de Vries (Rotterdam, The Netherlands) and Chen Zhou (De Nederlandsche Bank).

The main sponsor of the workshop is GENERALI Insurance.

The workshop is organized jointly by the following two FCT/MCTES research projects:

- *EXTREMA — STATISTICAL EXTREMES IN TODAY'S WORLD*, PTDC/MAT/101736/2008.
- *ENES — EXTREMES IN SPACE*, PTDC/MAT/112770/2009.

It is our hope that this event also strengthens the ties and encourages collaboration between researchers and practitioners in Statistics of Extremes, Insurance and Finance. We thank the authors for their prompt support with their interesting contributions. Finally, we must record our deep appreciation for the encouragement and support of José Alves, President of the Generali Insurance Company in Portugal, who had a prominent role in this event, under the auspices of the 100-years anniversary of the Faculty of Science of the University of Lisbon in 2011. Finally our sincere thanks go to all those involved in making this project successful.

M. Isabel **Fraga Alves**
M. Ivette **Gomes**
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# Contents

# 1

# *INVITED PAPERS*

# EXTREMAL DEPENDENCE OF TIME SERIES

**Holger Drees**
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**Abstract:** We consider time series of log returns of a financial investment. In order to assess the risk of extreme losses, it does not suffice to analyze the marginal tail behavior, because the potential total loss is strongly influenced by the clustering behavior of large negative returns on consecutive periods. We present a systematic approach to the analysis of the extremal serial dependence of such time series using empirical process theory. Particular attention is turned to the bias which is known to often cause serious misjudgment of the clustering behavior. Further potential applications of the theory, e.g. to the discrimination between time series models, are sketched.

# MATHEMATICAL PROBLEMS UNDERLYING QUANTITATIVE RISK MANAGEMENT

**Paul Embrechts**
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**Abstract:** QRM has become an important field of applied mathematical research with considerable impact in such fields as for instance Climate Change, Finance and Insurance. In this talk I will give examples of mathematical research resulting from QRM related questions. I will also give an outlook of potentially interesting future fields of methodological research.

# MODELING ELECTRICITY MARKETS: SPOTS, FORWARDS AND RISK PREMIUMS

**Claudia Klüppelberg**
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**Abstract:** We present a new model for the electricity market dynamics, which is able to capture seasonality, low-frequency dynamics and the extreme peaks in the spot price as well as the much less volatile forward prices. We introduce a non-stationary process for trend, seasonality and low-frequency dynamics, and model the large fluctuations by a non-Gaussian stable CARMA process. We identify all components of our model, in particular, we separate the different components of our model and suggest a robust $L_1$-filter to find the states of the CARMA process. We discuss possibilities for equivalent martingale measures in our heavy-tailed model, which leads to the estimation of the market price of risk and the risk premium in this market. We apply this procedure to data from the German electricity exchange EEX. For this market we detect a clear negative risk premium, which indicates that the electricity producers are price takers willing to accept a lower price to hedge their production. This is joint work with Fred Benth and Linda Vos from Oslo University.

# PRECISE LARGE DEVIATIONS PROBABILITIES FOR A HEAVY-TAILED RANDOM WALK

**Thomas Mikosch**
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**Abstract:** In this talk we will consider the tail probabilities of partial sum processes for stationary processes whose marginal distribution has power law tails. These results generalize the classical results by A.V. and S.V. Nagaev who showed that the "heavy-tail heuristics" applies in this case: the power law tails of the partial sums are essentially due to the maximum term in the sum; see Section 8.6 in Embrechts, Klüppelberg and Mikosch (1997). The situation changes in the case of dependent sequences. Then extremal clusters shape the form of the tails of the partial sums. But in contrast to the tails of the maxima, the extremal index does not appear in these quantities. In contrast to the tail behavior of partial maxima there are only very few particular cases where we can determine the tail behavior of partial sums for stationary sequences. We will consider some known cases, compare them with the iid case and indicate how these large deviation results can be used to proved results about ruin probabilities.

## References

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# RISK MEASURES OF AUTOCORRELATED HEDGE FUND RETURNS

**Casper de Vries**
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**Abstract:** Standard risk metrics tend to underestimate the true risks of hedge funds due to serial correlation in the reported returns. Getmansky, Lo and Makarov (2004) derive mean, variance, Sharpe ratio and beta formulas adjusted for serial correlation. Following their lead, we derive adjusted downside and global measures of univariate and systemic risk. We distinguish between normally and fat tailed distributed returns and show that adjustment is particularly relevant for downside risk measures in the case of fat tails. A hedge fund case study reveals that the unadjusted risk measures considerably underestimate the true extend of single and multivariate risks.

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1. Getmansky, M., Lo, A. and I. Makarov (2004), An econometric model of serial correlation and illiquidity in hedge fund returns. *Journal of Financial Economics* **74**, 529–609.

# SYSTEMIC RISK IN FINANCIAL SYSTEM: AN EXTREME VALUE APPROACH

**Chen Zhou**
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**Abstract:** The unfolding of the financial crisis since 2008 raises the questioning on the current regulation and supervision of the financial system. In the debate of regulation reform, instead of limiting risk taking of individual financial institution, managing systemic risk is widely agreed as the focus of the new regulation framework, the so-called “macro-prudential” regulation. We investigate the systemic risk on the cross-sectional dimension: the interconnectedness among financial institutions. This talk departures from comparing a few potential measures on the systemic risk, continues with discussing potential drivers driving the systemic risk, and concludes with policy advices that help manage the systemic risk. Extreme Value Theory, particularly its multivariate version, is the major tool in both theoretical modeling and empirical assessment within this context.

# *2*

# *CONTRIBUTED PAPERS*

# MODELLING CHANGES IN THE UNCONDITIONAL VARIANCE OF LONG STOCK RETURN SERIES[1]

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**Abstract:** This paper focus on the modelling of deterministic changes in the unconditional variance over a long time series. The unconditional variance is parameterized according to a slowly moving function in which the variance is allowed to evolve smoothly over time. For the purpose, we consider the multiplicative decomposition as in Amado and Teräsvirta (2011) for modelling the long-run volatility and the short-run dynamics volatility in the series. The parameterization of the long-run component is specified using a sequence of Lagrange multiplier-type tests. The model building procedure is illustrated with an application to the daily returns of the DJIA index covering a period of ninety years of financial market history. Two major conclusions are as follows. First, the LM tests strongly reject the assumption of constancy of the unconditional variance. Second, the results show how the long-memory property in volatility may be also explained by ignoring changes in the unconditional variance of the series.

## 1. Introduction

The issue that deterministic shifts in long return series can generate the observed long-memory property has received much attention in recent years.

[1]This research has been supported by the Danish National Research Foundation.

Most of the work in this topic is related with the study of the behaviour of standard statistical tools and model misspecification under nonstationarity. Early studies include Diebold (1986) and Lamoureux and Lastrapes (1990) who suggest that occasional level shifts in the intercept of the GARCH model can bias the estimation towards an integrated GARCH model. More recently, Mikosch and Stărică (2004) argue that the so-called 'integrated GARCH effect' is caused by the nonstationary behaviour of very long return series. They show how the long-range dependence in volatility and the IGARCH effect may be explained by neglected deterministic changes in the unconditional variance of the stochastic process. Moreover, Granger and Hyung (2004) claim that occasional breaks in a long time series of absolute stock returns can also explain the slow decay of the autocorrelation functions of absolute returns.

It is well documented that shocks to the conditional variance of the standard GARCH model of Bollerslev (1986) decay at an exponential rate having almost no influence for long time optimal forecasts. This has motivated the development of more flexible models to describe the observed dependence structure in financial market volatility. One of these models is the Fractionally Integrated GARCH model of Baillie *et al.* (1996) which belongs to the class of long-memory models. In these processes, shocks to the conditional variance decay at a slow hyperbolic rate which is more strongly supported by financial data than the GARCH model. A generalization of the FIGARCH model was recently proposed by Baillie and Morana (2009) in which they allow the intercept to change deterministically according to Gallant (1984)'s flexible functional form.

The question of modelling explicitly nonstationarity in stock market volatility has, however, received somewhat less attention. There have been some attempts to incorporate nonstationarity directly into the model. Stărică and Granger (2005) proposed a new nonstationary approach in which the returns are modelled as nonstationary sequence of independent random variables with time-varying unconditional variance but their model does not allow for volatility clustering. More recently, Engle and Rangel (2008) propose modelling the volatility process by a multiplicative decomposition into a nonstationary and a stationary component. The nonstationary component (or the unconditional variance) is described by an exponential spline, and the stationary component (or the short-run dynamics of volatility) follows a GARCH process.

This paper addresses the issue of modelling deterministic changes in the unconditional variance over a long return series. The model considered in this paper assumes that volatility is modelled by a multiplicative decomposition of both conditional and unconditional variance following Amado and Teräsvirta (2011). The conditional variance follows a GARCH-type process, and describes the short-run dynamics of volatility. The nonstationary component of volatility describes the long-volatility dynamics, and it is represented by a linear combination of logistic transition functions. Statistical inference is used for specifying the parametric structure of the time-varying component by applying a sequence of Lagrange multiplier tests. Our modelling strategy is applied to describe the long-run properties of the long Dow Jones Industrial Average (DJIA) return series from 1920-2010. One may expect that for longer sample period, the more likely the occurrence of structural changes or shifts in the second unconditional moment of the returns. The test results strongly support the time-variation of the unconditional variance in the period under study. The results indicate that the strongest deterministic changes in the unconditional variance are associated with the largest economic recessions which suggests that the unconditional variance behaviour may be related with the evolution of the deterministic conditions in the economy. Our findings also suggest that the observed long-memory property in volatility is mostly caused by deterministic changes in the unconditional variance of the return series.

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# DPOT METHODOLOGY – AN APPLICATION TO VALUE-AT-RISK[2]

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**Abstract:** Threshold methods, based on fitting a stochastic model to the excesses over a threshold, were developed under the acronym POT (peaks over threshold). In order to eliminate the tendency to clustering of violations, a model based approach within the POT framework, that uses the durations between excesses as covariates, is presented. Based on this approach, models for forecasting one-day-ahead Value-at-Risk were applied to real data. Comparative studies provide evidence that they can perform better than state-of-the art risk models and much better than the widely used RiskMetrics model.

## 1. Introduction

Investors and traders must pay attention not only to the expected return from their activities but also to the risks that they incur. It is widely accepted that risk-adjusted performance measures can guide institutions toward a better risk/return profile and can play a relevant role to achieve a more secure financial system. This justifies the interest of developing more accurate risk models. Value-at-Risk (VaR) has emerged as the standard measure in quantitative risk management. In terms of regulation, the Basel II Accord requires that Authorized Deposit-taking Institutions (ADIs) to report their daily VaR forecasts to the monetary authorities at the beginning of each trading day and defines daily capital requirements based on these forecasts. We will consider the symmetric of daily log returns, $R_{t+1} = -\log(V_{t+1}/V_t) \times 100$, where $V_t$ is the

[2]Research partially supported by Fundação para a Ciência e a Tecnologia (FCT/PROTEC, FCT/OE and FCT/PTDC).

value of the portfolio at time $t$. The one-day-ahead VaR forecast made at time $t$ for time $t+1$, $VaR_{t+1|t}(p)$, is defined by $P[R_{t+1} > VaR_{t+1|t}(p)|\Omega_t] = p$, where $\Omega_t$ is the information set up to time-$t$ and $p$ is the *coverage rate*. A *violation* occurs when $R_{t+1} > VaR_{t+1|t}(p)$. We review the peaks over threshold (POT) method with an example that illustrates the problem of *tendency to clustering of violations*. In order to solve this problem, we propose risk models based on *durations* and within the POT framework. Comparisons between the proposed risk models and other models are carried out.

# 2. A duration based POT method (DPOT)

Our main goal is to eliminate the tendency to clustering of violations that occurs with the POT method. To achieve this goal, within the POT framework we propose the presence of durations between excesses as covariates. Smith (1990), developed ML and Least Squares estimation procedures under the POT framework with the shape and scale parameters dependent on covariates. For a general overview of EVT and its application to VaR, including the use of explanatory variables, see, for instance, Tsay (2010). For details about the mathematical theory of EVT and its applications to risk management, see Embrechts *et al.* (1997).

Let $y_1,...,y_n$ be the excesses above a high threshold $u$, $d_1$ the duration until the first excess and $d_2,...,d_n$, defined by $d_i = t_i - t_{i-1}$, where $t_i$ denotes the day of excess $i$. We propose to use from the information set up to time t ($\Omega_t$), the last $v$ durations between excesses, $d_n, d_{n-1},...,d_{n-v+1}$ and the duration since the excess $n$ which we define by $d^t$. With the durations $d_i,...,d_{i-v+1}$, it is possible to consider at the time of excess number $i$, the duration since the preceding $v$ excesses, defined by $d_{i,v} = d_i + ... + d_{i-v+1} = t_i - t_{i-v}$. At day $t$, after the excess $n$, we define $d_{t,1} = d^t$, $d_{t,2} = d^t + d_n$ and for $v = 3,4,\cdots$, $d_{t,v} = d^t + d_{n,v-1} = d^t + d_n + \cdots + d_{n-v+2}$, which represents the duration until $t$ since the preceding $v$ excesses.

## 2.1. DPOT Model

With the durations $d_2,...,d_n$ and the duration since the excess $n$, $d^t$, we assume the GPD for the excesses $Y_i$ above $u$, such that

$$Y_t \sim GPD\Big(\gamma, \sigma_t = g(\alpha_1,...,\alpha_k,...,d^t,d_n,d_{n-1},...,d_{n-v+2})\Big),$$

where $\gamma, \alpha_1, ..., \alpha_k$, are parameters to be estimated. And we propose the following class of estimators

$$\widehat{\text{VaR}}_{t+1|t}^{DPOT}(p) = u + \frac{\hat{\sigma}_t}{\hat{\gamma}}\Big(\Big(\frac{n}{n_x p}\Big)^{\hat{\gamma}} - 1\Big),$$

with $\hat{\sigma}_t = g(\hat{\alpha}_1, ..., \hat{\alpha}_k, ..., d^t, d_n, d_{n-1}, ..., d_{n-v+2})$.

Empirical results suggested an inverse relation between excesses and durations since the preceding $v$ excesses, with $1/(d_{i,v})^c$, $c > 0$, which leads to the specification $\sigma_t = \alpha \frac{1}{(d_{t,v})^c}$ and the VaR estimator

$$\widehat{\text{VaR}}_{t+1|t}^{DPOT(v,c)}(p) = u + \frac{\hat{\alpha}}{\hat{\gamma}(d_{t,v})^c}\Big(\Big(\frac{n}{n_x p}\Big)^{\hat{\gamma}} - 1\Big),$$

where $\hat{\gamma}$ and $\hat{\alpha}$ are estimators of the parameters $\gamma$ and $\alpha$. The log likelihood is

$$-\sum_{i=v}^{n} \log\Big(\frac{\alpha}{(d_{i,v})^c}\Big) - \Big(\frac{1}{\gamma} + 1\Big)\sum_{i=v}^{n} \log\Big(1 + \frac{\gamma}{\alpha} y_i (d_{i,v})^c\Big).$$

We present results for $v = 3$, $c \in \{0.8, 0.75, 0.7\}$. Using the proposed models with the S&P 500 Index returns, we obtain for 14190 one-day-ahead VaR forecasts, 138 (0.9725%), 134 and 134 (0.9443%) violations, respectively with $c = 0.8$, $c = 0.75$ and $c = 0.7$. These percentages are much closer to the expected 1% than the 1.367% obtained with the unconditional POT model. For the 2008 global financial crises period, Figure 2.1 shows how the DPOT models solve the problem of tendency to clustering of violations, producing much better risk forecasts that adjust quickly to the high volatility in the returns during September and October.

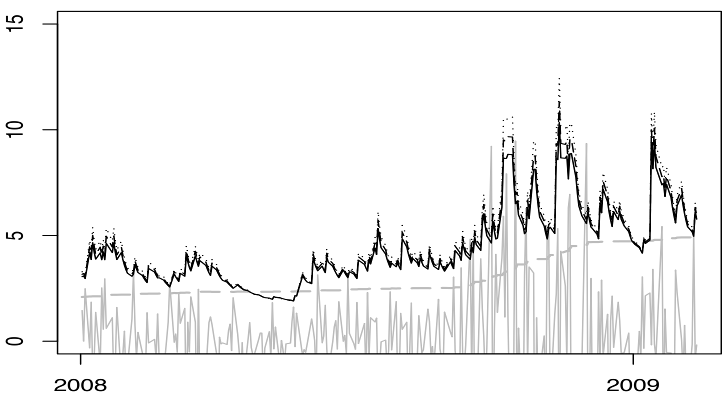


Figure 2.1: Symmetric returns of S&P 500 Index from January 2, 2008 through February 12,2009 (solid grey), and one-day-ahead VaR(0.01) forecasts with DPOT($c = 0.8$) (dotted), DPOT($c = 0.75$) (longdash), DPOT($c = 0.7$) (solid), POT method (longdash grey) and a rolling window of size 1000.

Within this period of 282 days, the number of violations with DPOT($c = 0.8$) was 8, with DPOT($c = 0.75$) was 8 and with DPOT($c = 0.7$) was 11, much less than the 29 violations obtained with the unconditional POT method. Empirical findings suggest that a choice of $c = 0.75$ is preferable. We also study the model with $c$ estimated, but we achieve poor results.

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# OPTIMIZATION OF PORT OPERATIONAL AND SAFETY[3]

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**Abstract:** This paper presents a model for the integral management of port activities in real-time. It considers the harbour system as a partition of the space and a set of chains of activities that are interrelated and exposed to the occurrence of different threats. The model jointly examines safety and operational levels by analyzing both failures and their propagation along the chains of activities, and estimating the associated risk.

## 1. Introduction

The transport of goods and passengers is one of the main sectors of the economy of a country in an increasingly globalized world. Particularly in Spain, it represents about 5% of the annual GDP and employs almost 6% of the population (Fundación Cetmo, 2005).
The EU has stimulated maritime transport in recent years and Spain has become a strategic area of maritime traffic because of its proximity to major trade routes. Actually, port activity represents more than 1% of the Spanish GDP, with 35,000 direct jobs and over 110,000 indirect jobs (Cendrero and Truyols, 2008).
In this context, it is essential that all port operations are conducted with the

[3]Research partially supported by the Ministerio de Fomento de España, projects 53/08 and CIT-46000-2009-21

maximum efficiency and safety. In addition to these two requirements, there are also two important features to take into account. On the one hand, the potential strong impact that the harbor system has in the surrounding environment. On the other hand, public funds that must be managed in a transparent manner.
There are many studies about risk and reliability of certain port activities and infrastructures (eg Dalsgaard *et al.*, 2000, Solari *et al.*, 2010). In most of them, safety and operational capacity are treated separately. These elements are analyzed without considering their relationship with other activities and parts of the port.
This paper proposes a model for the management of a port in real-time. The model considers the port system as a set of elements and chains of activities exposed to the occurrence of various threats. It analyzes potential failures and tracks them along the activity chains and parts of the port. Thus it simplifies the estimation of the consequences in monetary terms and in terms of damage to people and environment. The model allows to jointly analyze both harbour safety and operational levels. Furthermore, it is possible to assesses the risk associated to different threats and to check the performance of protocols designed to be activated in case of threat. To accomplish this goal the model must be properly fed with simulations of the values characterizing forcing agents.

# 2. Model description

The model is based on the discretization of the port in areas and the organization of harbour operations in chains of activities. These chains are classified into (1) main chains, that are defined according to each merchandise handled in the harbour, (2) secondary chains, the aim of which is to ensure that the activity is conducted safely, efficiently and within the regulatory framework, and (3) auxiliary chains which aim to ensure the welfare of harbour users.
Activities themselves are also classified attending to the objective they individually pursue. They are defined by, among other characteristics, their average duration, the areas where they are carried out, the monetary benefit per unit time and the components used. Components include the people involved and the necessary procedures to implement them.
Our model also distinguishes different types of threats. It considers their associated failure and stoppage modes. Lastly, it evaluates the protocols designed to mitigate threats consequences in case of occurrence.
Real-time port activity is simulated by taking as unit time the port state, which allows to describe its different parameters and their updating. Port state is for-

mally defined as the time interval in which any manifestation of the harbour system can be regarded as stationary from a statistical perspective, according to the ROM 0.0 (Losada, 2001).
The simulation includes the analysis of possible failures that affect harbour safety and its operational capacity. It also studies propagation along the chains and areas, and assess personal, environmental and economic consequences.

## 2.1. Implementation

The use of traditional forms of knowledge representation, such as relational databases, has not been found appropriate. This is due to (1) the large amount of information that is required to run the model, (2) the structure in which it has to be stored and (3) the variety of relationships between the components. Instead, we chose to use an ontology (Neches *et al.*, 1991), which allows a formal definition of the port system in a hierarchical and systematic way so that the information can be functionally manipulated as people would do it.

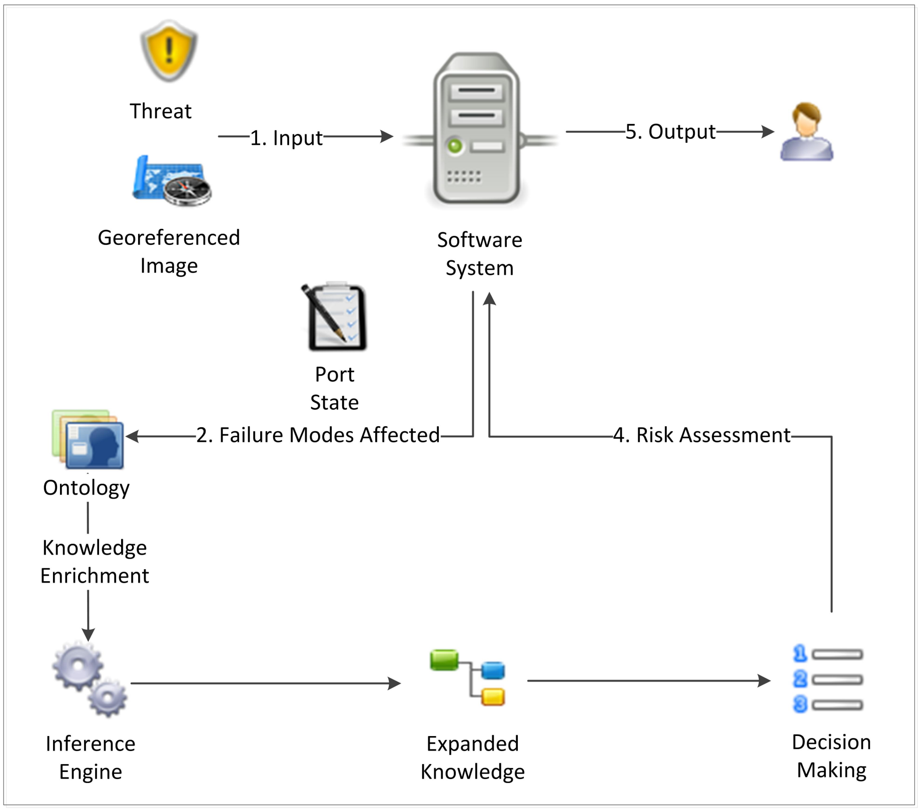


Figure 2.2: Model implementation

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# ASYMPTOTIC AND BOOTSTRAP CONFIDENCE BOUNDS FOR THE ADJUSTMENT COEFFICIENT[4]

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**Abstract:** In this work we consider the problem of estimating the adjustment coefficient $R$ in the Sparre Andersen model. Based on a geometric-type estimator for $R$, we construct confidence bounds using the tail bootstrap method and the first order of the Edgeworth expansion of the estimator. A simulation study is performed in order to analyse and compare the finite sample behaviour of these confidence bounds.

**Key words and phrases:** adjustment coefficient, Edgeworth expansions, Sparre Andersen model, tail index estimation.

## 1. Estimating the adjustment coefficient

Consider the Sparre Andersen model for claims arriving at an insurance company, and assume that the sequence $C_1, C_2, \ldots$ of claims occur at times $T_1, T_1 + T_2, \ldots$, where $\{C_i\}$ and $\{T_i\}$ are independent sequences of i.i.d. r.v.'s. Starting with initial capital $x$ and with incoming premiums in the time interval $[0,t]$ equal to $\gamma t$, the risk reserve is $S(t) = x + \gamma t - \sum_{i=1}^{N(t)} C_i$, where $N(t) = \max\{n \geq 0 : \sum_{i=1}^{n} T_i \leq t\}$ is the number of claims observed up to time t. The probability of ruin is then given by $U(x) = P\{\max_{n\geq 1} \sum_{i=1}^{n} (C_i - \gamma T_i) > x\}$.

Let $D_i = C_i - \gamma T_i$ $i = 1, 2, \ldots$ be i.i.d. r.v.'s with $E(D_1) < 0$. Assume also that

$$\exists R > 0 : E(e^{RD_1}) = 1 \text{ and } E(|D_1| e^{RD_1}) < \infty. \tag{2.1}$$

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The unique positive solution $R$ is called the *adjustment coefficient*. The importance of this coefficient in risk theory follows from the well-known Cramér-Lundberg inequality for the ruin probability: $U(x) \leq e^{-Rx}$, for all $x > 0$, and the asymptotic relationship: $U(x) \sim \lambda e^{-Rx}$, as $x \to \infty$, where $\lambda$ is a positive constant.

Csörgő and Steinebach (1991) suggested to estimate $R$ by means of a sequence of auxiliary r.v.'s $\{Z_k\}$, recursively defined as follows:

$$M_0 = 0,\ M_n = \max\{M_{n-1} + D_n, 0\} \quad \text{for } n = 1, 2, \ldots,$$
$$\nu_0 = 0,\ \nu_k = \min\{n \geq \nu_{k-1} + 1 : M_n = 0\} \quad \text{for } k = 1, 2, \ldots,$$
$$Z_k = \max_{\nu_{k-1} < j \leq \nu_k} M_j \quad \text{for } k = 1, 2, \ldots.$$

as $z \to \infty$. Csörgő and Steinebach observed that, if $C(t) := \sum_{i=1}^{N(t)} C_i$ is a compound Poisson process, or if the claims $C_i$ are exponentially distributed, then

$$P(Z_1 > z) = ce^{-Rz}\{1 + O(e^{-Az})\}$$

as $z \to \infty$, with positive constants $c$ and $A$.

More recently, Brito and Freitas (2006) have shown that, under (2.1),

$$P(Z_1 > z) = ce^{-Rz}(1 + o(1)) \text{ as } z \to \infty.$$

We are interested in estimating $R$, more precisely, in constructing asymptotically correct confidence bounds for $R$. We consider here the geometric-type estimator given by

$$\widetilde{R}(k_n) = \left( \frac{1}{k_n} \sum_{i=1}^{k_n} Z_{n-i+1,n}^2 - \left( \frac{1}{k_n} \sum_{i=1}^{k_n} Z_{n-i+1,n} \right)^2 \right)^{-1/2},$$

where $Z_{1,n} \leq Z_{2,n} \leq \ldots \leq Z_{n,n}$ denote the order statistics of the sample $Z_1, Z_2 \ldots, Z_n$ and $k_n$ is a sequence of positive integers satisfying

$$1 \leq k_n < n, \quad \lim_{n\to\infty} k_n = \infty \quad \text{and} \quad \lim_{n\to\infty} k_n/n = 0.$$

Using the results established in Brito and Freitas (2003) and Brito and Freitas (2006) we have that $\widetilde{R}(k_n)$ is asymptotically normal and the tail bootstrap method introduced by Bacro and Brito (1998) works for this estimator. In Brito and Freitas (2008) we derived an Edgeworth expansion for $\widetilde{R}(k_n)$, allowing the construction of accurate confidence bounds for $R$.

# 2. Confidence bounds

Consider the normalized estimator

$$R_n := \frac{R^2}{\sqrt{8}} k_n^{1/2} \left( \frac{1}{\widetilde{R}^2(k_n)} - \frac{1}{R^2} \right).$$

If the distribution of $R_n$ was known, we could calculate the corresponding $q$-quantile $x_q$, i.e $P(R_n \le x_q) = q$. Writing $p = 1 - q$, the lower $100p\%$ confidence interval for $R$ would be:

$$CI_{\widetilde{R}}(k_n, p) = \left( \widetilde{R}(k_n) \left( 1 + \sqrt{2} x_q k_n^{-1/2} + o(k_n^{-1/2}) \right), +\infty \right).$$

For estimating $x_q$ we may use the asymptotic normality of $R_n$, to obtain:

$$NI_{\widetilde{R}}(k_n, p) \quad = \quad \left( \widetilde{R}(k_n) \left( 1 + \sqrt{2} \Phi^{-1}(q) k_n^{-1/2} + o(k_n^{-1/2}) \right), +\infty \right).$$

Using the asymptotic Edgeworth expansion derived in Brito and Freitas (2008) we obtain more accurate interval estimates for $R$:

$$\begin{aligned} & EI_{\widetilde{R}}(k_n, p) \\ & = \left( \widetilde{R}(k_n) \left( 1 + \sqrt{2} \Phi^{-1}(q) k_n^{-1/2} - \sqrt{2} p_1(\Phi^{-1}(q)) k_n^{-1} + o(k_n^{-1}) \right), +\infty \right). \end{aligned}$$

We also consider the tail bootstrap confidence bounds given by:

$$BI_{\widetilde{R}}(k_n, p) \quad = \quad \left( \widetilde{R}(k_n) \left( 1 + \sqrt{2} \widehat{x}_q k_n^{-1/2} + o(k_n^{-1/2}) \right), +\infty \right),$$

where $\widehat{x}_q$ is the $q$-quantile of the tail bootstrap d.f. of $R_n$

As a consequence of the results obtained in Brito and Freitas (2006) and Brito and Freitas (2008), we have that these bounds give, under usual conditions, asymptotically correct coverage rates, that is,

$$P(R \in BI_{\widetilde{R}}(k_n, p)) \to p \text{ and } P(R \in EI_{\widetilde{R}}(k_n, p)) \to p, \text{ as } n \to \infty.$$

A small simulation will be performed in order to analyse and compare the finite sample behaviour of these confidence bounds.

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# PROBABILITY WEIGHTED MOMENTS BOOTSTRAP ESTIMATION: A CASE STUDY IN THE FIELD OF INSURANCE[6]

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**Abstract:** We make use of *probability weighted moments* of largest observations, in order to build classes of estimators of the *extreme value index*. Due to the specificity of the estimators, we propose the use of bootstrap computer intensive methods for an adaptive choice of the optimal number of order statistics to be used in the estimation. The methodology is applied to data in the field of insurance.

## 1. Introduction and preliminaries

The *extreme value index* (EVI) is the real parameter $\gamma$ in the general *extreme value* (EV) distribution function (d.f.), $G_\gamma(x) := \exp(-(1+\gamma x)^{-1/\gamma})$, $1+\gamma x > 0$. Let $\underline{X}_n = (X_1, \dots, X_n)$ denote a random sample of size $n$, and consider the associated sample of ascending order statistics (o.s.'s) $(X_{1:n} \le \cdots \le X_{n:n})$. One of the first classes of semi-parametric estimators of a positive EVI was the Hill (H) estimator ([4]), given by

$$\hat{\gamma}^H_{k,n} := \sum_{i=1}^{k} \{\ln X_{n-i+1:n} - \ln X_{n-k:n}\} / k, \tag{2.2}$$

for $k = 1, 2, \dots, n-1$. We shall also deal with the *Pareto probability weighted moments* (PPWM) EVI-estimators, recently introduced in [1]. They are valid for $0 < \gamma < 1$, compare favourably with the Hill estimator, and are given by

$$\hat{\gamma}^{PPWM}_{k,n} := 1 - \hat{a}_1(k) / \big(\hat{a}_0(k) - \hat{a}_1(k)\big), \tag{2.3}$$

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with $\hat{a}_0(k) := \sum_{i=1}^{k} X_{n-i+1:n}/k$ and $\hat{a}_1(k) := \sum_{i=1}^{k} (i/k)\, X_{n-i+1:n}/k$. Consistency of these EVI-estimators is achieved if $X_{n-k:n}$ is an *intermediate* o.s., i.e., if $k = k_n \to \infty$ and $k/n \to 0$, as $n \to \infty$. In order to derive the asymptotic normality of these EVI-estimators, and with the notation $U(t) := \inf\{x : F(x) \geq 1 - 1/t\}$, $t \geq 1$, it is often assumed the validity of a second-order condition, like $\lim_{t\to\infty} \big(\ln U(tx) - \ln U(t) - \gamma \ln x\big)/A(t) = (x^\rho - 1)/\rho$, where $|A| \in RV_\rho$, $\rho \leq 0$. Under such a second-order framework, if $\sqrt{k}A(n/k) \to \lambda_{_A}$, finite, as $n \to \infty$, these EVI-estimators are asymptotically normal. Denoting $\hat{\gamma}^\bullet_{k,n}$, any of the estimators above, we have, with $Z^\bullet_k$ asymptotically standard normal and for adequate $(b_\bullet,\ \sigma_\bullet) \in (\mathbb{R},\ \mathbb{R}^+)$,

$$\hat{\gamma}^\bullet_{k,n} \stackrel{d}{=} \gamma + \sigma_\bullet Z^\bullet_k/\sqrt{k} + b_\bullet\, A(n/k)(1 + o_p(1)), \quad \text{as} \quad n \to \infty. \tag{2.4}$$

After a review, in Section 2, of the role of the bootstrap methodology in the estimation of optimal sample fractions, we provide a reference to an algorithm for the adaptive estimation through the Hill estimators, also valid for the PPWM EVI-estimators. In Section 3, as an illustration, we apply such a data-driven estimation to a data set in the field of insurance.

# 2. The bootstrap methodology and optimal levels

Under the above mentioned second-order framework, with $\rho < 0$, let us use the parameterization $A(t) = \gamma\beta t^\rho$, where $\beta$ and $\rho$ are generalized scale and shape second-order parameters. Given the EVI-estimator, $\hat{\gamma}^\bullet_{k,n}$, let us denote $k_0^{\hat{\gamma}^\bullet}(n) := \arg\min_k \mathrm{MSE}(\hat{\gamma}^\bullet_{k,n})$, with MSE standing for *mean squared error*. With $\mathbb{E}$ denoting the mean value operator and AMSE standing for *asymptotic mean squared error*, a possible substitute for $\mathrm{MSE}(\hat{\gamma}^\bullet_{k,n})$ is

$$\mathrm{AMSE}(\hat{\gamma}^\bullet_{k,n}) := \mathbb{E}\big(\sigma_\bullet\, \overline{Z}_k/\sqrt{k} + b_\bullet A(n/k)\big)^2 = \sigma_\bullet^2/k + b_\bullet^2\, \gamma^2\, \beta^2\, (n/k)^{2\rho},$$

cf. equation (2.4). Then, with the notation $k_{0|\hat{\gamma}^\bullet}(n) := \arg\min_k \mathrm{AMSE}\big(\hat{\gamma}^\bullet_{k,n}\big)$, we get $k_{0|\hat{\gamma}^\bullet}(n) = k_0^{\hat{\gamma}^\bullet}(n)(1 + o(1))$. For the Hill estimator, we have, in (2.4), $(b_{_H}, \sigma_{_H}) = (1/(1-\rho), \gamma)$. Consequently, with $(\hat{\beta}, \hat{\rho})$ a consistent estimator of $(\beta, \rho)$ and $[x]$ denoting the integer part of $x$, we have an asymptotic justification for the estimator $\hat{k}_0^H := [((1-\hat{\rho})^2 n^{-2\hat{\rho}}/(-2\hat{\rho}\hat{\beta}^2))^{1/(1-2\hat{\rho})}] + 1$. The same does not happen with the PPWM EVI-estimators, due to the fact that $\sigma_{_{PPWM}}$ and $b_{_{PPWM}}$ depend both on $\gamma$. It is sensible to use the bootstrap methodology for the adaptive PPWM EVI-estimation. Similarly to what has been done in [3], for the

H estimator, we can use the algorithm in [2], considering the auxiliary statistic, $T^{\bullet}_{k,n} := \hat{\gamma}^{\bullet}_{[k/2],n} - \hat{\gamma}^{\bullet}_{k,n}$, $k = 2, \ldots, n-1$, which converges to the known value zero, and double-bootstrap it adequately on the basis of samples of sizes $n_1 = o(n)$ and $n_2 = [n_1^2/n]$, in order to estimate $k_{0|\hat{\gamma}^{\bullet}}(n)$, through a bootstrap estimate $\hat{k}^{\bullet}_{0*}$. Note also that bootstrap confidence intervals (CIs) are easily associated with the bootstrap EVI-estimates, through the replication of the above-mentioned algorithm $r$ times.

# 3. A case study

We shall next consider an illustration of the performance of the adaptive PPWM EVI-estimates under study, comparatively with the same methodology applied to the Hill EVI-estimates, again through the analysis of $n = 371$ automobile claim amounts exceeding 1,200,000 Euro over the period 1988-2001, gathered from several European insurance companies co-operating with the same reinsurer, Secura Belgian Re. The above-mentioned algorithm led us to $\hat{\rho}_0 = -0.74$ and $\hat{\beta}_0 = 0.80$. For a sub-sample size $n_1 = [n^{0.955}] = 284$, and $B = 250$ bootstrap generations, we were led to $\hat{k}^{PPWM}_{0*} = 58$ and to $PPWM^* = 0.272$. This same algorithm applied to the Hill estimates leads us to $\hat{k}^{H}_{0*} = 52$ and to $H^* = 0.299$.

In Figure 2.3, as a function of the sub-sample size $n_1$, ranging from $n_1 = [n^{0.95}] = 275$ until $n_1 = [n^{0.9999}] = 370$, we picture, at the left, the estimates $\hat{k}_{0*}(n_1)/n$ of the *optimal sample fraction* (OSF), $k^{\bullet}_0/n$, for the adaptive bootstrap estimation of $\gamma$ through the Hill and the PPWM estimators, in (2.2) and (2.3), respectively. Associated bootstrap EVI-estimates are pictured at the right. Contrarily to the bootstrap Hill, the bootstrap PPWM EVI-estimates are quite stable as a function of the sub-sample size $n_1$ (see Figure 2.3, *right*).

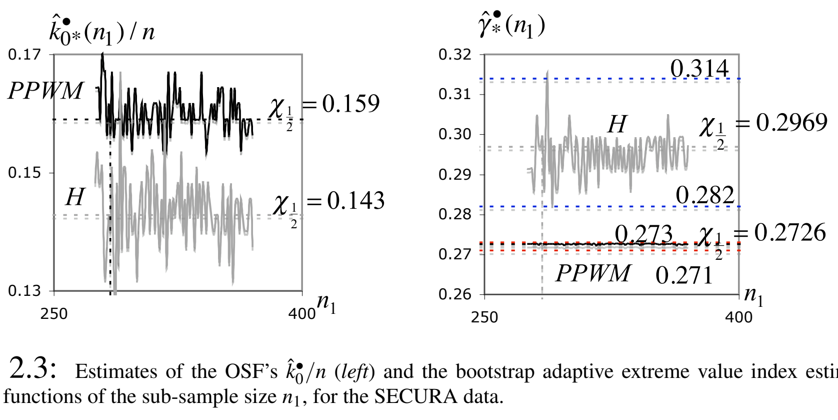


Figure 2.3: Estimates of the OSF's $\hat{k}^{\bullet}_0/n$ (*left*) and the bootstrap adaptive extreme value index estimates $\hat{\gamma}^{\bullet}_{*}$ (*right*), as functions of the sub-sample size $n_1$, for the SECURA data.

The running of the above mentioned algorithm $r = 100$ times, for $n_1 = [n^{0.955}]$, provided, for the PPWM-estimates, a median 0.2726, an average 0.2725, and a 95% bootstrap CI for $\gamma$ given by $(0.271, 0.273)$, as shown in Figire 2.3. The equivalent indicators for the bootstrap Hill estimates were 0.2969, 0.2949 and $(0.282, 0.314)$, also shown in Figure 2.3. The size of the CIs are in favour of the PPWM estimation. Indeed, the H-estimates are clearly overestimating the true value of the EVI, and should be used with care.

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# ASSESSING EXTREMAL DEPENDENCE IN EQUITY MARKETS

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**Abstract:** In recent years there has been an increasing interest in modelling dependence in heavy tail phenomena such as the latest turbulence episodes in financial markets. The evidence of asymptotic indepen- dence in the financial data has led to the need of rethinking risk modelling and inference tools for multivariate extremes. In this paper we propose an inference scheme for assessing extremal dependence of several pairs of variables, in the context of asymptotic independence. Our approach is based on the fact that the problem of interest can be rewritten as an empirical likelihood problem for comparing the means of different populations, where such means represent the Hill's estimate of the coefficient of tail dependence. A triangular array rep- resentation allow us to obtain a nonparametric Wilks' theorem, and we apply the method to assess extremal dependence in equity markets.

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# ON REFINING PRICING METHODS FOR LIABILITY INSURANCE CONTRACTS

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**Abstract:** This paper aims to combine the great benefits obtained by the Extreme Value Theory and the estimation of a claim size distribution for the entire set of actuarial data. Indeed, it is well known that the task of refining pricing methods for liability insurance crucially relies on the underlying claim size model. Our proposal is to estimate first the larger losses via the excess over a threshold technique, obtaining a tail with Pareto behaviour. Then, the second step is to combine this right tail with a more classical model for the bulk of the data, by extending some results in the recent literature.

## 1. Introduction

The usual practice of insurance companies, when modeling claim size distributions, is to fit classical models (often Lognormal and Pareto) to the data. However, when pricing high excess layer insurance contracts, it is critically important to obtain a good estimate for the right tail of the distribution. Extreme Value Theory (EVT) may be used for this purposes (see [2], [3], among many others). On the other hand, in some cases it is desirable to obtain a model able to give a good estimation of insurance losses over the entire range of values, to price both ground-up and excess of loss layers in a consistent way.

## 2. The Data Set: Public Liability Losses

The data used in this analysis is Public Liability (PL) insurance losses, experienced in some London local authorities from 1997 to 2002. The figures have been adjusted for inflation, to restate values at today's prices. In Figure 2.4 the data is shown in a time series graph and in a histogram using the log scale. The mean-excess plot is also shown. It can be clearly noted that there is a big concentration of losses under £50,000. Specifically, 92% of the claims are below

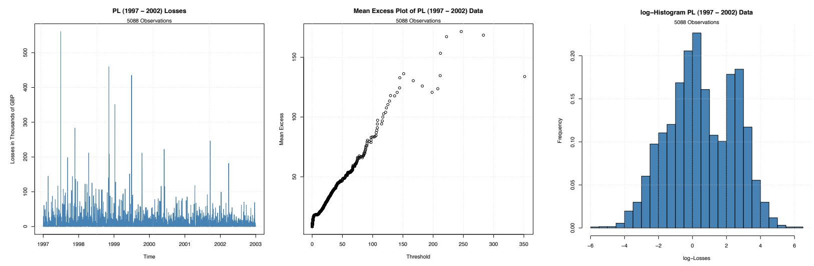


Figure 2.4: From left to right: data in time series graph, mean-excess plot and log-histogram of the data.

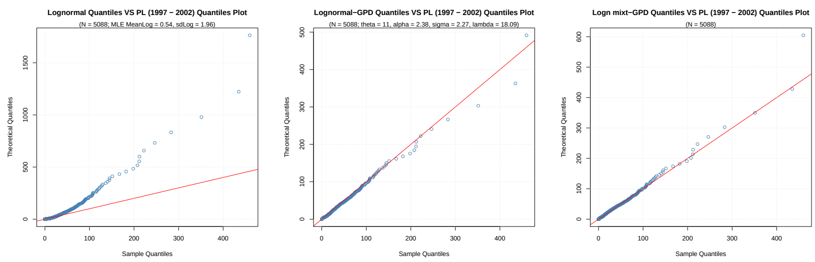


Figure 2.5: *QQ* plot of the three models, obtained via ML method, against the empirical quantiles of the PL losses. Note the correspondence for the highest quantiles when using the EVT approach.

£25,000. The histogram in log scale seems to take the well known bell shape, suggesting a Lognormal fit for the majority of the data. However, there is a second peak on the right side, which indicates that claim sizes are generated from overlapping processes.

# 3. Composition of EVT and Classical Models

The proposal is to model with a Generalized Pareto Distribution (GPD) exactly the fraction $(1-r)$ of largest losses which exhibits this asymptotic behaviour, by adopting the threshold $\theta$ (the minimum value of the loss over which a roughly linear behaviour of the mean-excess function can be observed) and the tail index $\xi$ estimated by the EVT analysis. As a second step, for the left $r$ part of the data (the lower losses), a Lognormal distribution is considered initially. The idea is to combine the two models, requiring continuity and derivability in the transition point $\theta$, using some recent results due to [4]. In a second stage, due to the bimodality of the original data set, a fit has been obtained by combining a mixture of Lognormals with the GPD.

The *QQ* plots in Figure 2.5 show the improvement given by the composite models. The second model offers a conservative estimation for the highest quantiles, while the third can be preferred because of some doubts con-

| **Model** | $A_1$ | $A_2$ | $A'_2$ |
|---|---|---|---|
| **Lognormal** | 1.02 | 1.42 | 1.58 |
| **Lognormal-GPD** | 0.23 | 0.28 | 0.35 |
| **Lognormal mixture-GPD** | 0.13 | 0.16 | 0.16 |

Table 2.1: Goodness of fit measures for the three fits. The improvement obtained by the (second and) third model is apparent. For the use of these index into this context see [1]

| **Layer No.** | ***Limit*** | ***LL*** | ***UL*** | ***C (Lognormal)*** | ***C (Logn-GPD)*** | ***C (Logn mixt-GPD)*** |
|---|---|---|---|---|---|---|
| **Basic** | 5 | 0 | 5 | 2.384 | 2.497 | 2.445 |
| **Excess1** | 5 | 5 | 10 | 1.148 | 1.239 | 1.417 |
| **Excess2** | 10 | 10 | 20 | 1.372 | 1.359 | 1.749 |
| **Excess3** | 30 | 20 | 50 | 1.960 | 1.434 | 1.738 |
| **Excess4** | 100 | 50 | 150 | 2.122 | 0.831 | 0.910 |
| **Excess5** | 100 | 150 | 250 | 0.776 | 0.159 | 0.191 |
| **Excess6** | 150 | 250 | 400 | 0.570 | 0.081 | 0.108 |

Table 2.2: Cost per claim indications C, calculated by fitting the Lognormal, the Lognormal-GPD and the Lognormal mixture-GPD model. All the figures are in £1,000 units. The first contract covers a ground-up basic loss from £0 to £5,000. *Excess1* can be read as "£5,000 excess of £5,000".

cerning the way the oldest claims are adjusted to today prices. To measure the goodness-of-fit, the Mortara index $A_1$, the quadratic K. Pearson index $A_2$ and the modified quadratic index $A'_2$ have been used (see Table 2.1). For the Lognormal-GPD composite model, $A'_2 = 0.35$ means there is still some discrepancy between observed and modeled frequencies, greatly reduced by the third model.

# 4. Price Indications

Just to show a simple application of our results, let $L$ be the amount of a single loss to an insurance contract. The insurance payment $S$ will be *L* - *LL*, if the loss $L$ is bigger than the lower limit *LL*, but the payment is limited to *Limit* = *UL* - *LL*, if $L$ is larger than the upper limit *UL*. The expected loss $C$ for the excess of loss contract arising from one claim can be calculated as

$$C = \mathbf{E}L = \int_{LL}^{UL} (x - LL) f(x) dx + Limit \int_{UL}^{\infty} f(x) dx.$$

In Table 2.2 the cost of claim is computed using the Lognormal distribution with ML parameters $\mu = 0.54$ and $\sigma = 1.96$ (mean and standard deviation of the log-transformed RV), the Lognormal-GPD model with ML parameters, according to the notation given in [4], $\sigma = 2.27$, $\theta = 11$, $\xi = 0.42$ and $\lambda = 18.09$ and the Lognormal mixture-GPD model with ML parameters $k = 0.81$ (the weight of the first component of the mixture), $m_1 = 0.047$ and $m_2 = 12.151$ (the two modes), $\sigma_1 = 1.740$, $\sigma_2 = 0.617$, $\theta = 36.23$ and $\lambda = 4.319$.

# 5. Conclusions and Further Developments

To summarize, we have used historical data to fit loss distributions, by taking into account the entire set of the data. We combined EVT and classical models in order to refine pricing methods for liability insurance contracts. The data involved in the analysis suggested that a mixture of Lognormal distributions may be used for modeling the lower losses, hence extending the approach of [4]. This model reflects better the nature of the PL insurance business, where attritional losses generated by personal injury claims and large losses due to property damages can result in multiple density peaks. Also, the paretian tail is often a good model for property losses. A number of questions are still open, the assessment of model uncertainty and parameter uncertainty by means, say, of bootstrap techniques. The estimation of the frequency distributions of the losses needs to be incorporated and we are currently involved in further developments.

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# THE INFLUENCE OF MINIMUM CAPITAL LEVELS IN FINANCIAL RISK

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**Abstract:** We address recent directives from Basel III concerning the raise of minimum capital levels of banks on an individual basis, with the aim of lowering the probability for a large crash to occur. Using an agent-based model, we present evidence that in such new financial reality large losses due to extreme events are avoided only if one assumes that banks will accept quietly the drop of business levels. Assuming that banks will try to restore business levels, raising diversification, the probability of large crisis does not necessary decrease and may even increase in certain cases, due to the complexity of the underlying network.

# A NEW CHARACTERISTIC FOR THE DEPENDENCE STRUCTURE OF CLUSTERED EXTREMES

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**Abstract:** We will question the informative value of some well-known cluster characteristics for financial time series concentrating on the GARCH model in particular. To this end, we will discuss typical questions about the structure of extremes and their implications for the management of risks with respect to the extremal index and the extremal coefficient function. Our reasoning will give rise to the proposal of a novel summary measure for the extremal dependence of time series. We will evaluate this measure for the GARCH(1,1) class.

## 1. Extremal Clusters of GARCH Processes

Two stylized facts of financial returns, namely volatility clustering and heavy tails, can be modelled simultaneously by the class of GARCH processes. The analysis of extremal clusters of these processes calls for simple measures that basically indicate the main characteristics of such events. Here, the extremal index that essentially corresponds to the average cluster size, and the extremal coefficient function, an analog of the autocovariance for extremes, cf. Schlather and Tawn (2003), have been proposed. For both characteristics no analytical expression in terms of the GARCH parameters exists. A simulation based evaluation of both measures has been studied in Mikosch and Stărică (2000), Laurini and Tawn (2006) and Fasen *et al.* (2010).

Our contribution is twofold. First, we make use of the fact that the extremes of a Markov chain may be modelled by a certain (back and forth) tail chain, cf. Segers (2007). Its behavior virtually corresponds to a random walk given an extreme observation of the original process as a starting point. The tail chain approach has been implemented in de Haan *et al.* (1989) with respect to the extremal index of ARCH processes. We show that it also applies to the GARCH(1,1) family and substantially simplifies a number of previous considerations of this kind. Second, we shall discuss why reflection invariance may be considered a shortcoming of the extremal coefficient function. This, of course, depends on the respective application but it appears to be reasonable to require an appropriate measure for the within extremal cluster structure to address the following questions:

(Q1) What is the probability for a second, third etc. extreme event occuring two, three etc. days after the outset of a financial crisis?

(Q2) What is the structure of a cluster of high-level exceedances and how does the course of extreme events (i.e. the evolution of a stress period or crisis over time) typically look like?

(Q3) How may we characterize the memory spread of financial markets with respect to shocks, i.e. how long does a crisis typically last?

Consequently, we find that many possible questions appear to focus on expected events in the near future given a *first* extreme event today (i.e. the beginning of a temporary shock or crisis). In practice, e.g. for financial institutions, the expected location of such events within a cluster of extremes is essential in order to efficiently react to the pattern of inherent risk they describe.

## 2. A New Cluster Characteristic

For a stationary stochastic process $X = (X_t, t \in \mathbb{Z})$ let the extremal coefficient function be given by

$$\phi(h) = \lim_{x\to\infty} P(X_h > x \mid X_0 > x), \quad h \in \mathbb{Z},$$

i.e. the probability of two extreme events separated by a lag $h$. Following the above arguments, however, the first extreme event in a cluster is often of special practical interest. We shall therefore propose a modification of the extremal coefficient function given by

$$\gamma(h) = \lim_{x\to\infty} P(X_h > x \mid X_0 > x, X_{-1} \le x, X_{-2} \le x, \ldots), \quad h \in \mathbb{N},$$

that indicates the probability of extreme events following the outset of a crisis at lag $h$. We discuss some major differences of the properties for $\phi(h)$ and $\gamma(h)$. In particular, note that $\gamma(h)$ is not reflection invariant. Our evaluation of $\gamma(h)$ for a GARCH(1,1) process therefore relies on both the back and forth behavior of the respective tail chain.

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# ADAPTIVE REDUCED BIAS ESTIMATION OF FINANCIAL LOG-RETURNS[7]

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**Abstract:** Jointly with a set of classical extreme value index (EVI) estimators, we suggest the consideration of associated second-order corrected-bias estimators, and propose the use of resampling-based methods for an asymptotically consistent choice of the *thresholds* to use in an adaptive EVI-estimation of financial log-returns.

## 1. CVRB EVI-estimators under study

We shall deal with the estimation of a positive *extreme value index* (EVI), denoted $\gamma$, the primary parameter in *Statistics of Extremes*. Apart from the classical Hill (Hill, 1975) and moment (Dekkers *et al.*, 1989) semi-parametric EVI-estimators, based on the largest $k$ top order statistics and denoted $H(k)$ and $M(k)$, respectively, we shall also consider associated classes of second-order reduced-bias estimators, in the lines of Gomes *et al.* (2011). These classes are based on the adequate estimation of a "scale" and a "shape" second-order parameters, $\beta \neq 0$ and $\rho < 0$, respectively, are valid for a large class of heavy-tailed underlying parents and are appealing in the sense that we are able to reduce the asymptotic bias of a classical estimator without increasing its asymptotic variance. We shall call these estimators "*classical-variance reduced-bias*" (CVRB) estimators. The CVRB class associated with $H(k)$ was introduced in

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Caeiro *et al.* (2005) and it is given by

$$\overline{H}(k) := H(k)\big(1 - \hat{\beta}(n/k)^{\hat{\rho}}/(1-\hat{\rho})\big),$$

being a *minimum-variance reduced-bias* (MVRB) class of EVI-estimators. Associated with $M(k)$, we have the CVRB EVI-estimator,

$$\overline{M}(k) := M(k)\big(1 - \hat{\beta}\,(n/k)^{\hat{\rho}}\,/(1-\hat{\rho})\big) - \hat{\beta}\;\hat{\rho}\,(n/k)^{\hat{\rho}}\,/(1-\hat{\rho})^2.$$

Let us generally denote $C$ any of the classical $H$ and $M$ estimators, and $\overline{C}(k)$ any of the reduced-bias estimators. Under the validity of adequate third-order conditions, and adequate estimation of $(\beta,\rho)$ (see Caeiro *et al.*, 2009; Gomes *et al.*, 2011), $\overline{C}(k)$ outperforms $C(k)$, $\forall k$.

In Section 2, we briefly refer an adaptive EVI-estimation based on bootstrap methods, similar in spirit to the bootstrap adaptive classical EVI-estimation in Gomes and Oliveira (2001), and references therein, and to the bootstrap adaptive MVRB estimation in Gomes *et al.* (2009). In Section 3, we refer partial results of a Monte-Carlo simulation related with the behaviour of the non-adaptive estimators. Finally, in Section 4, we provide an application to the analysis of log-returns of a financial time series.

## 2. Adaptive classical and CVRB EVI-estimation

With AMSE standing for "asymptotic mean square error (MSE)", $\hat{\gamma}$ denoting either $C$ or $\overline{C}$, and with $k_0^{\widehat{\gamma}}(n) := \arg\min_k MSE(\widehat{\gamma}(k))$, we again get $k_{0|\widehat{\gamma}}(n) := \arg\min_k AMSE(\widehat{\gamma}(k)) = k_0^{\widehat{\gamma}}(n)(1+o(1))$, and a double bootstrap based on subsamples of size $n_1 = o(n)$ and $n_2 = [n_1^2/n]$ enabled Gomes *et al.* (2011) to consistently estimate the optimal sample fraction of $\overline{C}(k)$, on the basis of a consistent estimator of $k_{0|\widehat{\gamma}}(n)$. Such a double bootstrap leads to a $k_0$-estimate $\hat{k}^*_{0\hat{\gamma}}$ and to an adaptive EVI-estimate, $\widehat{\gamma}^* := \widehat{\gamma}(\hat{k}^*_{0\hat{\gamma}})$. In order to obtain a final adaptive EVI-estimate on the basis of one of the estimators under consideration, we still suggest the estimation of the MSE of any of the EVI-estimators at the bootstrap $k_0$-estimate, denoted $\widehat{MSE}(\hat{k}^*_{0\hat{\gamma}}|\hat{\gamma}^*)$, and the choice of the estimate $\hat{\gamma}^{**} := \arg\min_{\hat{\gamma}^*} \widehat{MSE}(\hat{k}^*_{0\hat{\gamma}}|\hat{\gamma}^*)$.

## 3. A Monte-Carlo simulation

Comparatively with the behaviour of the classical EVI-estimators $H(k)$ and $M(k)$, we next illustrate, in Figure 2.6, the finite-sample behaviour of the

CVRB EVI-estimators, $\overline{H}(k)$ and $\overline{M}(k)$, providing the patterns of mean values (E) and root mean square errors (RMSE) of the estimators, as a function of $h = k/n$, for an underlying Fréchet parent, and sample sizes $n = 500$. Similar results have been obtained for other simulated models. Note the clear reduction in bias achieved by any of the reduced-bias estimators. Such a bias reduction leads to lower mean square errors for the CVRB estimators, comparatively with the associated classical EVI-estimators.

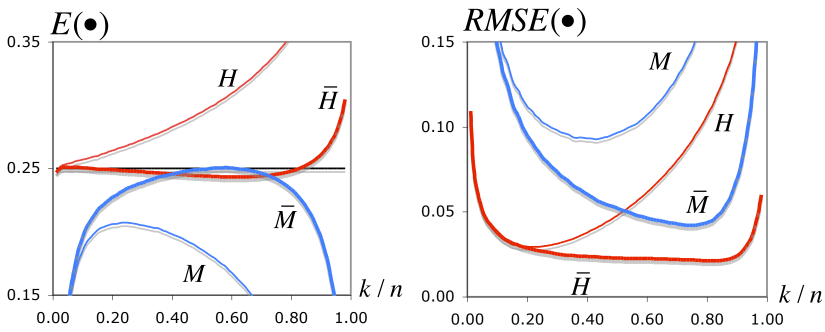


Figure 2.6: Patterns of mean values (*left*) and root mean square errors (*right*) of the classical estimators $H$ and $M$, jointly with the associated CVRB estimators, as functions of $k/n$, for an underlying Fréchet parent with $\gamma = 0.25$ $(\rho = -1)$.

# 4. An application to financial data

For the daily log-returns of IBM, collected from January 4, 1999, until November 17, 2005 (with a size $n = 1762, n^+ = 881$), we show in Figure 2.7, the sample paths of $C(k)$ and $\overline{C}(k)$, for $C = H$ and $M$, jointly with the bootstrap adaptive EVI-estimates described in Section 2. The results clearly favour the $\overline{C}$-estimators. We have been led to the estimate $\overline{H}^{**} = 0.364$.

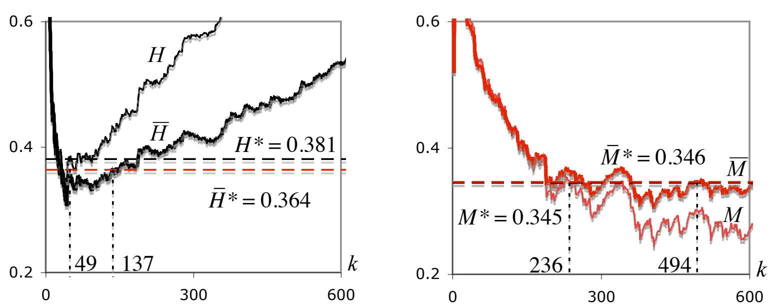


Figure 2.7: Estimates of $\gamma$, through the EVI-estimators under consideration, for the IBM log-returns.

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# LIQUIDITY RISK AND SOLVENCY II

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**Abstract:** This paper discusses the importance of liquidity risk when evaluating the risk of portfolios of financial assets that insurance companies hold. Until very recently and within the scope of Solvency II, liquidity risk was only considered under Pillar II, i.e. the proposal was that insurance companies should perform a mere qualitative evaluation of it. Nowadays the possible quantitative evaluation of liquidity risk is under debate but it is still unclear if it will apply only to liabilities or to portfolio holdings as well. The authors argue that liquidity is an important source of market risk and that it should be measured quantitatively when accessing the overall market risk of portfolio holdings. Based upon the financial liquidity literature, they propose a way to measure liquidity risk quantitatively. The proposed method is simple, relies on publicly available data, and is consistent with the VaR approach underlying Solvency II. This paper implements the proposed method on the Portuguese insurance sector, using actual portfolio holdings. The main empirical findings confirm that liquidity risk is an important risk representing, on average, more than 10% of the overall market risk insurance companies are exposed to.

## 1. Introduction and Motivation

The recent financial crisis and subsequent turmoil in financial markets have sparked new questions about the perception and evaluation of liquidity risk. This discussion has taken place in the context of a new regime that aims to supervise and regulate insurance and reinsurance in the European Union, the Solvency II Directive, which will come into force by the end of 2012. The period preceding the economic and financial crisis witnessed extraordinary economic performance worldwide, with never before seen growth rates and strong gains in the stock markets. Record low interest rates led to a boom in the residential housing markets, and this coincided with the development and proliferation of new, innovative structured credit products. When several credit institutions in the US holding subprime mortgages went bankrupt, it was rumoured that there could be significant losses in hedge funds which had invested in products that were based on subprime mortgages. As a result, investors became concerned about the evaluation of all financial credit instruments, and ratings agencies began to downgrade the ratings of some companies and structured financial products. The problems in the credit markets spread to other markets, leading to generalized lack of confidence and consequent lack of liquidity in financial markets, the banks raise their spread rates significantly, and this in turn led to the collapse of some major investment banks. This credit and liquidity crisis in the banking sector quickly spread to the insurance sector, to the extent that the largest insurance company in the world at the time faced severe liquidity problems. The situation was brought about by its investment in credit default swaps and the huge number of policy holders wishing to surrender policies that involved financial insurance due to their loss of confidence in the financial capacity of the company. In such a context, with the world struggling to get out of the recent credit and liquidity crisis it is puzzling the debate on how to evaluate risks insurance companies are exposed to, seemed to overlook the importance of a proper evaluation of liquidity risk. In fact, it was not until very recently, in its March 2010 report, that the Committee of European Insurance and Occupational Pensions Supervisors (CEIOPS) first recognized the importance, at least for liabilities, of a quantitative evaluation of liquidity risk (under Pilar I). Before that report, liquidity risk was considered only under Pilar II and the recommendation was a mere qualitative evaluation.In this study we, therefore, focus in measuring liquidity risk of portfolio holdings and propose a concrete way to take liquidity risk into account when accessing the market risk insurance companies are exposed to, in the context of Pilar I of Solvency II.

Liquidity risk is one of the most important risks to affect the solvency of insurance companies. Simply put, it reflects the available resources and capacity of the insurer to manage the financial flows to ensure that the company is able to meets its responsibilities when they fall due. Regardless of the reasons for which an insurance company may be called upon to pay out, the simple fact that much of its equity is invested in securities, which cannot be readily converted into cash, constitutes a risk. Further, because this kind of liquidity risk is directly associated with the holding of investment portfolios by insurance companies, it should be seen as an integral part of the market risk of portfolio holdings. [4] looks at liquidity risk as a component of market risk. According to [2], market liquidity can be described in terms of the magnitude of the bid/ask spread, market depth, i.e., the volume of assets that can be traded without distorting the current market prices, and market resilience, i.e., the time taken for the price of a certain asset to return to its initial pre-traded value. [1] consider that the price of an asset includes not only the risk stemming from fluctuations in price, interest rates and exchange rates, but also liquidity risk - exogenous and endogenous.

# 2. Method and Main Empirical Findings

The purpose of this study was to use liquidity risk to quantify market risk and to help to determine the consequent capital requirements in accordance with the Solvency II Directive. To do so we wanted to use a simple method that would make use of easily available information and that would require no alterations to the methodologies already used to quantify conventional market risks. The method proposed by [1] provided us with such a tool as it uses bid/ask spread information and makes use of simple addition.

We applied the method to the portfolio holdings of 45 Portuguese insurance companies, which are subject to supervision by the Instituto de Seguros de Portugal (Portuguese Insurance and Pension Funds Supervisory Authority) using data from the close of the 2009 financial year. From the results obtained for a time interval of one year and a confidence level of 99,5%, it was found that on average, liquidity risk per operator was around 0,4%, and it accounted for 10,3% of the total adjusted market risk. In 40% of the insurance companies liquidity risk was greater than 10% of the total adjusted market risk, even though most of these insurance companies showed a below market average VaR. The results of this study are limited by the fact that only quoted financial assets for which information was available and consistent were used. As a result, it is highly likely that both liquidity risk and market risk have been underestimated. It is possible to adjust the liquidity of these assets by applying linear regression models, which are estimated in function of the characteristics and indicators of a group of bonds and financial instruments  market benchmarks  that are defined for each class of asset. Even though underestimated, the values found by means of this empirical analysis of the Portuguese insurance sector clearly indicate that it is worthwhile to include liquidity risk in the measurement of market risk.

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# AN EFFICIENT PEAK-OVER-THRESHOLD IMPLEMENTATION FOR OPERATIONAL RISK CAPITAL COMPUTATION[8]

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The purpose of this paper is operational risks quantification following the Basel Advanced Measurement Approach. One proposal of the Basel committee binds banks to carry out their own models on internal data sets to evaluate amounts of capital necessary to face these risks. In this paper, we focus on the computation of the loss distribution function (LDF) ([6]) which is currently used to evaluate this kind of risks ([2]). We compute it as a convolution of two distribution functions, one modeling the frequencies of the losses and the other one their severity. We restrict to a Poisson distribution for the frequencies, and we focus on the estimation of the severity distribution. As soon as all the fittings of classical distributions fail (using a Kolmogorov - Smirnov test), we decide to model it as a mixing of two distributions: one fitted on the most important losses using the Generalized Pareto Distribution (GPD) for which we need to estimate a threshold above which we fit the distribution, ([9], [5]), and another one on the remaining data considering a lognormal distribution. In this paper, we deeply discuss the threshold estimation, and we show that the methods used to estimate all the parameters of GDP impact drastically the computation of the VaR measure and then the corresponding capital requirement. This last point leads us to analyze in details the building of the information data sets we used, provided by the bank[9] following Basel

[8]Research partially supported by BPCE.

[9]We used data from the French Caisse d'Epargne perimeter.

Committee requirements. We point out that the structure of the information set can be a source of errors for the computation risks associated to the operational risks. The data set we use is organized into the Basel Matrix. In its first level of granularity, this matrix is made up of 56 cases - 8 business lines ("b") $\times$ 7 event types ("e"). Nevertheless, each event type might be decomposed in several elements. For example, the "external fraud" event may be shared in two items - "Theft and Fraud" and "Systems Security" (second level of granularity). In a third level, the element "Theft and Fraud" may be split in several components: "Theft/Robbery", "Forgery" and "Check kiting". After a deep analysis, we observe that the kind of losses expected from a fraud with a credit card does not correspond to losses caused by someone hacking the system for instance; nevertheless they are in the same case. Therefore, considering the largest level of granularity, we could face multimodal empirical distributions. Consequently, the methods used to model the losses depend on the granularity level choice. This choice might have a tremendous impact on capital requirement computations. Besides, we face a trade-off between quantity of data and robustness of the estimations: indeed, if the quantity of data is not sufficient, we cannot go lower in the granularity; on the another hand the ensuing empirical distribution is therefore an aggregate of various natures of data and the estimation of this last empirical distribution can be source of unusable results.

The methodology used to build the Loss Distribution Function (LDF) is very simple and efficient. We recall its representation. The LDF $G_{b,e}$ (whose density is denoted $g_{b,e}$) is a mixture of two distributions, the frequency distribution $p_{b,e}$ and the severity distribution $F_{b,e}$ (whose density is $f_{b,e}$):

$$G_{b,e} = \sum_{\gamma=0}^{\infty} p_{b,e}(\gamma;\bullet) F_{b,e}^{\otimes\gamma}(x;\bullet),\, x > 0, \tag{2.5}$$

with $G_{b,e} = 0, x = 0$, where $\otimes\gamma$ is the $\gamma$-order operator of convolution, and $g_{b,e} = \sum_{\gamma=0}^{\infty} p_{b,e}(\gamma;\bullet) f_{b,e}^{\otimes\gamma}(x;\bullet)$, $x > 0$. The severity distribution will be defined as a combination of a lognormal distribution on the center, and a GPD on the right tail whose density $f(x;u,\beta,\xi)$ is :

$$f_{b,e}^{GPD}(x;u,\beta,\xi) = \begin{cases} \frac{1}{\beta}\left(1+\xi\frac{x-u}{\beta}\right)^{-1-\frac{1}{\xi}}, \text{ if } x \geq u,\, 1+\xi\left(\frac{x-u}{\beta}\right) > 0,\, \beta > 0 \\ \frac{1}{\beta}\exp\left(-\frac{x-u}{\beta}\right), \text{ if } x \geq u,\, \xi = 0 \end{cases}$$

where $u \in \mathbb{R}^{*+}$ is the threshold. $\beta \in \mathbb{R}^{*}$ is the scale parameter and $\xi \in \mathbb{R}$ the shape parameter. Thus, the density of the severity distribution is:

$$f_{b,e}(x;u,\beta,\xi,\mu,\sigma)=\begin{cases} f_{b,e}^{(center)}(x;\mu,\sigma), \text{ if } x<u \\ f_{b,e}^{(tail)}=\frac{1}{1-\int_0^u f_{b,e}^{(center)}(x;\mu,\sigma)dx}\times f_{b,e}^{GPD}(x;u,\beta,\xi)), \\ \hfill \text{if } x\geq u, \end{cases} \tag{2.6}$$

where, $\mu$ and $\sigma$ are the lognormal distribution parameters.

In order to estimate the parameters of the distribution (2.6), we begin with the threshold $u$ carrying out a bootstrap method ([7], [3]) for which we give practical solutions. Once $u$ is found, $\xi$ and $\beta$ have to be estimated, therefore we implement a method based on the Anderson-Darling statistics ([8]). Then, assuming a lognormal distribution to model the central part of the severity distribution, we implement an Expectation-Maximization algorithm ([4]) to estimate the parameters of the distribution. Finally, the $\lambda$ parameter of the Poisson distribution $p_{b,e}(\gamma;\lambda)$ is estimated using maximum likelihood method. As soon as we have estimated the whole set of parameters, we can build the loss distribution function (2.5) using a convolution method based on a modified Monte Carlo Algorithm. As soon as the target of this paper is the computation of capital requirements, we compute the Value-at-Risks ([1]) enforced at 99.9%, based on the computation of the loss distribution function that we apply on real operational risks data sets.

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# ADAPTIVE REDUCED BIAS INVARIANT ESTIMATION OF A HEAVY RIGHT TAIL: AN APPLICATION TO FINANCIAL LOG-RETURNS[10]

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**Abstract:** In this paper, we discuss an algorithm for an adaptive estimation of the *extreme value index*, $\gamma$, the main parameter in the field of *statistics of extremes*. We suggest the use of bias-corrected estimators applied to a sample of excesses over a central order statistic, jointly with the use of the bootstrap methodology for the choice of the *tuning parameters* involved in the adaptive estimation of $\gamma$.

# 1. Introduction and the Hill estimator

Slightly more restrictively than working in the whole domain of attraction (for maxima) of the *extreme value* (EV) distribution function (d.f.), $EV_\gamma(x) := \exp\big(-(1+\gamma x)^{-1/\gamma}\big),\ 1+\gamma x > 0,\ \gamma \in \mathbb{R}$, usually denoted $\mathscr{D}_{\mathscr{M}}\big(EV_\gamma\big)$, we shall consider a positive *extreme value index* (EVI), i.e. we shall work in $\mathscr{D}^+_{\mathscr{M}} := \mathscr{D}_{\mathscr{M}}\big(EV_\gamma\big)_{\gamma>0}$. With the notation $F^{\leftarrow}(y) := \inf\{x : F(x) \geq y\}$ for the generalized inverse function of $F$, and $RV_\alpha$ for the class of regularly varying functions at infinity with an index of regular variation $\alpha$, equivalently to say that $F \in \mathscr{D}^+_{\mathscr{M}}$, we can say that as $t \to \infty$, $U(t) := F^{\leftarrow}\big(1-1/t\big) \in RV_\gamma,\ \gamma > 0$. For these heavy-tailed models and given a sample $\underline{\mathbf{X}}_n = (X_1, \ldots, X_n)$, the classical EVI-estimators are Hill estimators (Hill, 1975), denoted $H_{k,n} \equiv H(k; \underline{\mathbf{X}}_n)$, the average of the log-excesses, $\ln X_{n-i+1:n} - \ln X_{n-k:n},\ 1 \leq i \leq k < n$. Further details can be found in Embrechts *et al.* (1997) and de Haan and Ferreira (2006).

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# 2. PORT EVI-estimation

The Hill estimators are invariant for changes in scale, but they are not invariant for changes in location. Moreover, they suffer drastic changes when we change the location of an underlying model $F$. This led Araújo Santos *et al.* (2006) to introduce the so-called PORT EVI-estimators, with PORT standing for *peaks over random threshold*. These PORT estimators are functionals of a sample of excesses over a random threshold $X_{[nq]+1:n}$, i.e., functionals of

$$\underline{\mathbf{X}}_n^{(q)} := \left(X_{n:n} - X_{[nq]+1:n}, \ldots, X_{[nq]+2:n} - X_{[nq]+1:n}\right),$$

with $0 < q < 1$ (the random threshold is an empirical quantile), being still valid in $\mathscr{D}_{\mathscr{M}}^{+}$. If we think on the Hill estimators, the PORT-Hill estimators are given by

$$H_{k,n}^{(q)} := H(k; \underline{\mathbf{X}}_n^{(q)}),\ 0 < q < 1,\ 1 \le k < n_q := n - [nq] - 1.$$

# 3. MVRB EVI-estimation

The non-null asymptotic bias of the Hill estimators at optimal level and a rate of convergence of the order of $1/\sqrt{k}$ lead to Hill sample paths with a high variance for small $k$, a high bias for large $k$, and a very sharp mean square error (MSE) pattern, which makes difficult the estimation of $k_0^H := \arg\min_k MSE(H_{k,n})$. These are the main reasons for the introduction of reduced-bias EVI-estimators. For technical reasons, it is then usual to restrict $\mathscr{D}_{\mathscr{M}}^{+}$, assuming that, with $\gamma$, $C > 0$, $\rho < 0$, and $\beta \neq 0$,

$$U(t) = Ct^{\gamma}\left(1 + A(t)/\rho + o(t^{\rho})\right),\ A(t) = \gamma\beta t^{\rho},$$

i.e. the slowly varying function $L_U(t) = t^{-\gamma}U(t)$ ultimately approximates a finite non-null constant value. A simple class of *minimum-variance reduced-bias* (MVRB) is the one in Caeiro *et al.* (2005), denoted $\overline{H} \equiv \overline{H}_{k,n}$. Such a class depends on the estimation of the vector $(\beta, \rho)$ of second-order parameters. Its functional form is

$$\overline{H}_{k,n} \equiv \overline{H}_{k,n}(\hat{\beta}, \hat{\rho}) \equiv \overline{H}_{\hat{\beta},\hat{\rho}}(k; \underline{\mathbf{X}}_n) := H_{k,n}\left(1 - \hat{\beta}(n/k)^{\hat{\rho}}/(1 - \hat{\rho})\right),$$

where $(\hat{\beta}, \hat{\rho})$ needs to be an adequate consistent estimator of $(\beta, \rho)$, so that $\overline{H}_{k,n}$ performs better than $H_{k,n}$ for all $k$. Algorithms for such a kind of estimation are provided in Gomes and Pestana (2007), among others. Regarding the adaptive

MVRB EVI-estimation: in a way similar to the one used for the classical EVI-estimators, now with the use of an auxiliary statistic like

$$T_n^{\overline{H}}(k) := \overline{H}_{[k/2],n} - \overline{H}_{k,n},$$

$k = 2,\dots,n-1$, the bootstrap methodology in Gomes *et al.* (2009, 2011) can help us to estimate $k_0^{\overline{H}}(n) := \arg\min_k MSE\big(\overline{H}_{k,n}\big)$, for a class of models such that

$$U(t) = Ct^{\gamma}\big(1 + A(t)/\rho + \beta' t^{2\rho} + o(t^{2\rho})\big),\ A(t) = \gamma\beta t^{\rho},$$

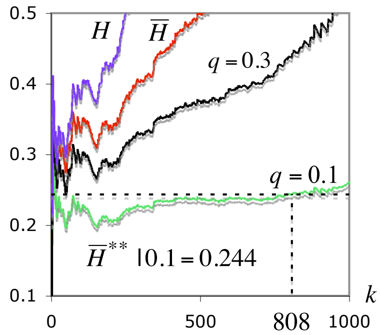
with $C, \gamma > 0$, $\beta, \beta' \neq 0$, $\rho < 0$.

# 4. Adaptive PORT-MVRB EVI-estimation: an application to financial data

We now suggest the use of the PORT-MVRB EVI-estimators, with the same functional form of $\overline{H}_{\hat\beta,\hat\rho}(k;\underline{\mathbf{X}}_n)$, but applied to the sample of excesses $\underline{\mathbf{X}}_n^{(q)}$. More precisely, on the basis of the PORT-Hill EVI-estimators, $H_{k,n}^{(q)}$, PORT-$\rho$, $\hat\rho_q \equiv \hat\rho^{(q)}$, and PORT-$\beta$, $\hat\beta_q \equiv \hat\beta^{(q)}$, we consider the class of estimators

$$\overline{H}_{k,n}^{(q)} = H_{k,n}^{(q)}\big(1 - \hat\beta_q (n/k)^{\hat\rho_q}/(1-\hat\rho_q)\big), \quad 1 \le k < n_q.$$

Note that with the notation $X_{n+1:n} = 0$, we can even consider that, by convention, $\overline{H} \equiv \overline{H}_{k,n}^{(q)}$, for $q = 1$, working thus with $q \in [0,1]$. The algorithm can be seen in Gomes and Henriques-Rodrigues (2010). On the basis of the computation of $\overline{H}_{k,n}^{(q)}$, $1 \le k < n_q$, for $q = 0.05(0.05)0.5, 1$, and the application of a double-bootstrap associated with sub-samples of size $m_1 = [n_q^{0.955}]$ and $m_2 = [n_1^2/n_q] + 1$, the algorithm leads us to estimates $\hat k_0^{(q)}$, and to EVI-estimates $\overline{H}_{n,m_1|T}^{*(q)} := \overline{H}_{\hat k_0^{(q)},n}^{(q)}$. We can next compute an estimate $\widehat{MSE}(k;q)$, find the optimal $q$-value, $\hat q := \arg\min_q \widehat{MSE}(\hat k_0^{(q)};q)$, and the adaptive final EVI-estimate, $\overline{H}^{**} \equiv \overline{H}^{**}|\hat q \equiv \overline{H}_{n,m_1|T}^{*(\hat q)} := \overline{H}_{\hat k_0^{(\hat q)},n}^{(\hat q)}$. For the log-returns associated to the daily close values of Microsoft Corp. (MSFT) stock, collected from January 4, 1999, until November 17, 2005 (with a size $n = 1762, n^+ = 882$), we were led to $\hat q = 0.1$, $\hat k_0^{(\hat q)} = 808$ and $\overline{H}^{**} = 0.244$, as shown in the following figure.

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# SOJOURN TIMES ABOVE A HIGH THRESHOLD FUNCTION

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**Abstract:** We investigate the sojourn time above a high threshold of a continuous stochastic process $\boldsymbol{Y} = (Y_t)_{t\in[0,1]}$ on $[0,1]$. It turns out that the limit, as the threshold increases, of the expected sojourn time given that it is positive, exists if the copula process corresponding to $\boldsymbol{Y}$ is in the functional domain of attraction of an extreme value process. This limit coincides with the limit of the fragility index corresponding to $(Y_{i/n})_{1\le i\le n}$ as $n$ and the threshold increase. If the process is in a certain neighborhood of a generalized Pareto process, then we can replace the constant threshold by a general threshold function and we can compute the asymptotic sojourn time distribution. An extreme value process is a prominent example.

## 1. Preliminaries

We call a process $\boldsymbol{\eta} \in C^- := \{f \in C[0,1] : f < 0\}$ a *standard* extreme value process (EVP), if it is a max-stable process (cf. de Haan and Ferreira [5]) with standard negative exponential (one-dimensional) margins, $P(\eta_t \le x) = \exp(x)$, $x \le 0$, $t \in [0,1]$.

It turns out (cf. [1]) that a stochastic process $\boldsymbol{\eta} \in C^-$ is a standard EVP if and only if there exists a number $m \ge 1$ and a stochastic process $\boldsymbol{Z} \in \bar{C}^+ = \{f \in C[0,1] : f \ge 0\}$ with the properties $\max_{t\in[0,1]} Z_t = m$ and $E(Z_t) = 1$ for all $t \in [0,1]$ such that for all $f \in \bar{E}^-[0,1]$ the equality

$$P(\boldsymbol{\eta} \le f) = \exp\left(-\|f\|_D\right) := \exp\left(-E\left(\sup_{t\in[0,1]} \left(|f(t)|Z_t\right)\right)\right) \tag{2.7}$$

holds. Thereby, $\bar{E}^-[0,1]$ denotes the set of all bounded real-valued functions on $[0,1]$ which are discontinuous at a finite set of points and which do not attain positive values.

We call $\boldsymbol{Z}$ the *generator process* of $\|\cdot\|_D$, where $\|\cdot\|_D$ in turn is called *D-norm* on $E[0,1]$.

In complete accordance with the unit- and multivariate case, we say that a process $\boldsymbol{V} \in \bar{C}^-$ is a *standard generalized Pareto process* (GPP), if there exists a $D$-norm $\|\cdot\|_D$ on $E[0,1]$ and some $c_0 > 0$ such that

$$W(f) := P(\boldsymbol{V} \le f) = 1 - \|f\|_D =: 1 + \log(G(f))$$

for all $f \in \bar{E}^-$ with $\|f\|_\infty \le c_0$, where $G(f) = P(\boldsymbol{\eta} \le f)$ is the "distribution function" of the EVP $\boldsymbol{\eta}$ with $D$-norm $\|\cdot\|_D$.

The main issue of functional EVT as introduced in Aulbach *et al.* [1] is the generalisation of the convergence underlying the functional domain of attraction idea: instead of weak convergence of probability measures on $C[0,1]$ as in de Haan and Lin [6] the convergence of "distribution functions" is considered.

We say that a stochastic process $\boldsymbol{Y} \in C[0,1]$ is *in the functional domain of attraction of a standard EVP* $\boldsymbol{\eta}$, denoted by $\boldsymbol{Y} \in \mathscr{D}(\boldsymbol{\eta})$, if there are functions $a_n \in C^+$, $b_n \in C[0,1]$, $n \in \mathbb{N}$, such that

$$\lim_{n\to\infty} P\left(\max_{1\le i\le n} \frac{\boldsymbol{Y}_i - b_n}{a_n} \le f\right) = \lim_{n\to\infty} P\left(\frac{\boldsymbol{Y} - b_n}{a_n} \le f\right)^n = \exp\left(-\|f\|_D\right) = G(f)$$

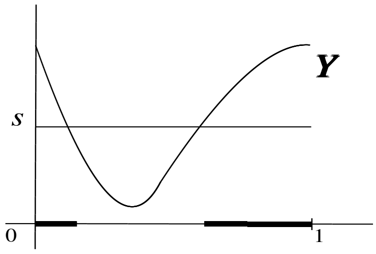
for any $f \in \bar{E}^-[0,1]$, where $\boldsymbol{Y}_1, \boldsymbol{Y}_2, \ldots$ are independent copies of $\boldsymbol{Y}$.

For example we get $\boldsymbol{U} \in \mathscr{D}(\boldsymbol{\eta})$ for a copula process $\boldsymbol{U} \in C[0,1]$ (i.e. all one-dimensional margins $U_t$ are uniformly distributed on [0,1]) if and only if $P(\boldsymbol{U} - 1 \le |c| f) = 1 + c\|f\|_D + o(c)$, $f \in \bar{E}^-[0,1]$, as $c \uparrow 0$.

# 2. Sojourn times

Let $\boldsymbol{Y} = (Y_t)_{t\in[0,1]} \in C[0,1]$ be a stochastic process with continuous sample paths and define

$$S(s) := \int_0^1 1(Y_t > s)\, dt,$$




wich is the *sojourn time* of $\boldsymbol{Y}$ above the constant treshold function $s \in \mathbb{R}$ (cf. [2] and the literature therein).

For arbitrary $n \in \mathbb{N}$ consider the n-dimensional random vector $(Y_{i/n})_{1 \le i \le n}$, i.e. a n-variate finite dimensional distribution of $\boldsymbol{Y}$.

Put $S_n(s) := \frac{1}{n}\sum_{i=1}^{n} 1(Y_{i/n} > s)$, which is the Riemann-sum of $S(s)$, and define $FI_n(s) := E(nS_n(s) \mid S_n(s) > 0)$.

The latter can for increasing $s$ be identified as the *fragility index* corresponding to $(Y_{i/n})_{1 \le i \le n}$ (cf. Geluck *et al.* [4]).

We establish the following result [3, Theorem 3.2]:

**Proposition 1:** *Let $\boldsymbol{Y} \in C[0,1]$ be a stochastic process with identical continuous marginal distribution function $F$. Suppose for the corresponding copula process $(F(Y_t))_{t \in [0,1]} =: \boldsymbol{U} \in \mathscr{D}(\boldsymbol{\eta})$, where $\boldsymbol{\eta}$ is a standard EVP with generator constant $m \ge 1$. Then*

$$\lim_{s \nearrow} E(S(s) \mid S(s) > 0) = \lim_{n\to\infty} \lim_{s \nearrow} \frac{FI_n(s)}{n} = \lim_{s \nearrow} \lim_{n\to\infty} \frac{FI_n(s)}{n} = \frac{1}{m}.$$

*Moreover:*

$$P(S(s) > 0) = (1 - F(s))m + o(1 - F(s)) \text{ as } s \nearrow \quad \text{and } E(S(s)) = 1 - F(s).$$

□

If $\boldsymbol{V} \in \bar{C}^{-}[0,1]$ is a standard GPP we can state the distribution of the sojourn time above an arbitrary threshold function $sf(t)$, $f \in \bar{E}^{-}[0,1]$, instead of the constant $s$:

**Proposition 2:** *Let $\boldsymbol{Z}$ the generator process of the GPP $\boldsymbol{V}$ and choose $f \in \bar{E}^{-}[0,1]$. Then there is an $s_0 > 0$ such that the sojourn time df $H_f$ of $\boldsymbol{V}$ above $sf$ is given by*

$$P\left(\int_0^1 1\left(V_t > sf(t)\right) dt > y \mid \int_0^1 1\left(V_t > sf(t)\right) dt > 0\right)$$
$$= \frac{\int_0^{m\|f\|_\infty} P\left(\int_0^1 1\left(|f(t)| Z_t > u\right) dt > y\right) du}{\int_0^{m\|f\|_\infty} P\left(\int_0^1 1\left(|f(t)| Z_t > u\right) dt > 0\right) du} =: 1 - H_f(y),$$

$0 \le y \le 1$, $0 < s \le s_0$, *provided the denominator is greater than zero.* □

This result can be extended to processes in a certain neighborhood of a GPP, for example a standard EVP: the asymptotic (conditional) sojourn time distribution for $s \downarrow 0$ can be computed (cf. [3, Proposition 4.4]).

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# ROBUST TOOLS FOR OPERATIONAL RISK[11]

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**Abstract:** We combine extreme value and robust statistics for quantifying the operational risk of a bank, modeled with a generalized Pareto distribution. To this end, we apply advanced robust approaches for parameter estimation and present diagnostics to assess the estimation quality at real operational loss data.

# 1. Introduction

According to the Pickands-Balkema-de Haan Theorem (PBdHT) [1, 6], events over a high threshold are approximately distributed according to a generalized Pareto distribution. This theorem may be applied in modeling of unexpected severe events, and is broadly used in operational risk measuring, i.e. for calculation of regulatory capital.

Following the Loss Distribution Approach, as in [2], this capital is quantified as a 99.9% quantile of a respective loss distribution. This quantile is a tail event, so one may indeed appeal to the PBdHT.

Estimation of GPD tail quantiles is not easy due to its heavy tail. This task gets even more difficult in operational risk applications by the fact that for some observations one cannot exclude that they are singular outliers, the reproducibility of which is questionable and, hence, their value for inference is doubtful. In addition, due to lack of historic data of a particular bank, data is pooled from several sources, which raises questions of representativeness. For these reasons, we apply robust methods in order to bound the individual influence of observations on inference.

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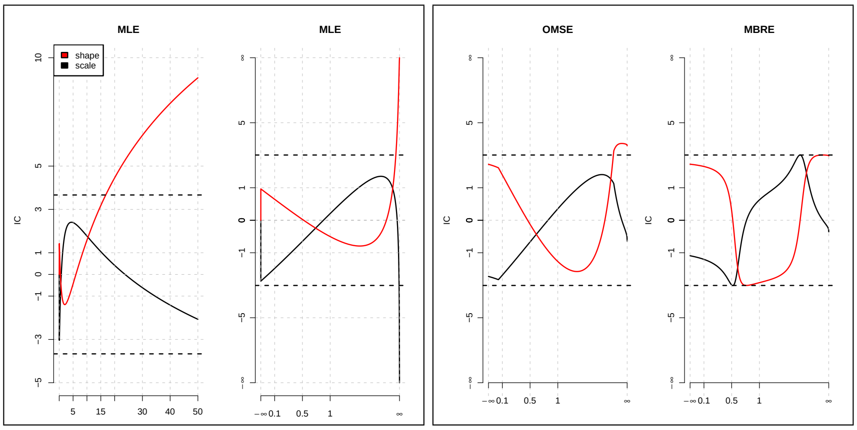


(a) Influence function of MLE in original and probit scales on the $y$-axis and logarithmic resp. quantile scales on the $x$-axis

(b) Influence functions of optimally-robust estimators in probit scale on the $y$-axis and quantile scale on the $x$-axis

Figure 2.8: Influence functions of MLE, MBRE and OMSE. Dashed black lines point out the bias of MBRE.

Robust approaches for operational risk have been discussed in Chernobai and Rachev [3]. Within robust statistics, our approach is based on shrinking neighborhoods and follows Huber-Carol [4], Rieder [7]. In this approach, one can distinguish optimally-robust procedures in quite general smooth parametric models, i.e.; the most bias robust estimator (MBRE), which minimizes the maximal bias of an estimator on such a neighborhood, and the optimal MSE robust estimator (OMSE), which minimizes the maximal mean squared error in the same setting. Both improve the classical maximum likelihood estimator (MLE) already in the presence of minor contaminations.

These optimally-robust estimators are computed as one-step reweighting procedures, and to this end need reliable starting estimators. Such a reliable estimator in the GPD model is given by the MedkMAD estimator. It takes up the general concept of L(ocation)D(ispersion) estimators from [5], which obtain estimates matching population location and dispersion quantities against their empirical counterparts. MedkMAD uses the median for the location part and an asymmetric variant of the median of absolute deviations for the dispersion part, and possesses good computational and global robustness properties [8].

ideal situation:

| estimator | \|Bias\| | Var | MSE | eff | rank | NA | time |
|---|---|---|---|---|---|---|---|
| **MLE** | 0.55 ±0.05 | 7.41 ±0.21 | 7.72 ±0.21 | 1.00 | **1** | 3.60 | 113 |
| MedkMAD | 0.71 ±0.07 | 11.96 ±0.31 | 12.46 ±0.30 | 0.62 | 3 | 0.79 | 223 |
| MBRE | 0.80 ±0.09 | 19.39 ±0.53 | 20.03 ±0.52 | 0.39 | 4 | 0.79 | 38 |
| OMSE | 0.95 ±0.07 | 11.36 ±0.34 | 12.25 ±0.33 | 0.63 | 2 | 0.79 | 41 |

contaminated situation:

| estimator | \|Bias\| | Var | MSE | eff | rank | NA |
|---|---|---|---|---|---|---|
| MLE | 394.12 ±22.92 | 1.37e7 ±1.20e6 | 1.52e7 ±1.37e6 | 0.00 | 5 | 3.61 |
| MedkMAD | 2.23 ±0.09 | 19.23 ±0.50 | 24.21 ±0.67 | 0.91 | 3 | 3.03 |
| MBRE | 1.77 ±0.09 | 20.06 ±0.59 | 23.19 ±0.63 | 0.95 | 2 | 3.03 |
| **OMSE** | 2.75 ±0.07 | 14.39 ±0.42 | 21.93 ±0.61 | 1.00 | **1** | 3.03 |

Table 2.3: Bias, variance, MSE and computational cost of MLE and optimally-robust estimators.

# 2. Experimental Results

We study MBRE and OMSE estimators in our GPD setting and compare them against the classical MLE. The influence functions of these estimators are plotted in Figure 2.8. We further calculate the estimators' asymptotic efficiency in both ideal and contaminated situations and study their finite sample behaviour in a simulation study. Finally, we provide diagnostics to assess the respective estimation quality applying the estimators to data from Algorithmics Inc.

# 3. Conclusion

Our empirical and theoretical results, briefly listed in Table 2.3, show that optimally-robust estimators, in praticular OMSE, are more preferable on data with outliers. As operational loos data is prone to contain outliers, we recommend to use robust methods instead of classical maximum likelihood.

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# AGGREGATION OF MARKET RISKS USING PAIR-COPULAS[12]

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**Abstract:** The advent of the Internal Model Approval Process within Solvency II and the desirability of many insurance companies to gain approval has increased the importance of some topics such as risk aggregation in determining overall economic capital level through the calculation of a risk measure, the Value-at-Risk ($99.5^{th}$VaR). The most currently used approach for aggregating risks is the variance-covariance matrix approach. Although being a relatively well-known concept that is computationally convenient, linear correlations fail to model every particularity of the dependence pattern between risks. In this paper we apply a pair-copula based model for aggregating market risks in order to calculate the economic capital needed to withstand both expected and unexpected future losses. This capital will be determined by computing a yearly $99.5^{th}$VaR.

# 1. Introduction

Insurance companies face a multitude of risks that could cause financial losses. Economic capital is the amount of capital that is needed to cover losses at a certain risk tolerance level. It captures a wide range of risks such as financial, insurance and operational risks and expresses all of this as a single number. Recently, Solvency II addressed risk aggregation as one of the main challenge in risk management. The most commonly used method is the variance-covariance approach which is described in section 2. While being easily implemented and convenient to work with, this approach can lead to a misestimation of the aggregated economic capital. Hence, a copula-based approach is chosen to achieve

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our purpose. However, one major shortcoming of copulas is their use in high dimensions. Indeed, elliptical copulas can be extended to higher dimension, but they are unable to model financial tail dependences (Patton, 2009), and Archimedean copulas are not satisfactory to describe multivariate dependence in dimensions higher than two (Joe, 1997).
The purpose of this work is to apply pair-copulas for aggregating market risks held by an insurance company and to compute then the $99.5^{th}$VaR.

# 2. Risk aggregation methodologies

## 2.1. Variance-Covariance Matrix Approach

The variance-covariance approach consists in computing the economic capital at the level of each risk and then aggregating these capitals through a matrix of correlation coefficients that reflect the degree of linear dependence between pairs of random variables.The entire economic capital can thus be determined through the following formula:

$$VaR_p(\alpha) = \sqrt{\sum_{i,j} \rho_{i,j} VaR_i(\alpha) VaR_j(\alpha)}$$

where $VaR_i(\alpha) = \omega_i.\sigma_i.F^{-1}(\alpha)$ is the stand alone VaR for risk i, $\omega_i$ the exposure, $\sigma_i$ its volatility, $F^{-1}(\alpha)$ the $\alpha^{th}$ quantile of the standardized portfolio losses and $\alpha$ the confidence level of the aggregate VaR. The $\rho$'s are the linear correlations between the risk pairs. Albeit this closed-form approach can be easily implemented, it suffers from ellipticity assumption and relies on the implicit constrain that the quantiles of the portfolio are the same as the quantiles of the marginal distributions, which is true for instance where all distributions are normal, but not when some of the risks have fat tailed distributions.This can yield a misestimation of the economic capital.

## 2.1. Copula-based Approach

The method we have chosen is to apply pair-copulas. While the variance-covariance method only requires measurement of the sub-risks economic capital estimates and the correlation between the sub-risks, copula aggregation methods depend on the whole distribution of the sub-risks. However, the method of extending two-dimension copulas to higher dimensions is not very flexible and additional assumptions are often needed. To bypass this problem, we use a pair-copula approach, also called vines: see Bedford and Cooke

(2002), Aas *et al.* (2006 and 2008), Guégan and Maugis (2010). The basic idea behind the pair-copula construction is to decompose an arbitrary distribution function into simple bivariate building blocks that are two-dimensional copulas and stitch them together appropriately. This method is straightforward to implement and can produce multivariate distribution functions of any dimension. For practical reasons, we consider in this paper two competitive methods when possible: nested copula models and vines.

# 3. Carrying out

Five proxies that represent the five main assets composing market risk are considered from January 1, 1999 to March 31, 2009: the daily Euro Stoxx index for Equity risk, the daily Euro 10 year-IR swap rates for interest rates risk, the daily Euro spread figures for spread risk, the daily Euro Implied Volatility rates for Implied volatility risk and the daily USD/EUR Foreign Exchange rates for FX risk. Each data set is filtered by applying an ARMA-GARCH process using GED innovations. We then determine the dependence structure between these five proxies applying the pair-copula methodology. Two kinds of dependence prevail: the Student-t copula and the Clayton one. We eventually compute the $99.5^{th}$ VaR from the previous modelings and determine the corresponding economic capital.

In order to be in line with Solvency II requirements, we only present the VaR figure. Nonetheless, the method can be extended to other risk measures such as the expected shortfall. Note that this preliminary work has never been performed before for aggregating market risks within insurance companies while these risks usually represent an important part of their risk profile. Until now, most of the risks have been aggregated using classical procedures and with this work we introduce a modeling of the dependence pattern between all market proxies by using a new approach - say vine structure - to answer to regulators.

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# CONDITIONAL ANALYSIS FOR MULTIVARIATE EXTREME RISK[13]

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**Abstract:** This poster discusses some recent theoretical development in the area of multivariate extreme value theory, based on the model proposed in Heffernan and Tawn (2004). The first half of the poster focuses on some of the unsolved issues in the original paper, namely the self-consistency issue and the residual distribution. The second half of the poster explores the an application of Heffernan-Tawn model in risk management and proposes a factor-based hierarchical multivariate extreme value model.

## 1. Introduction

This section starts by reinforcing the importance of extreme risk management in the financial industry and moves onto a quick introduction to the background of multivariate extreme value theory. This is then followed by an overview of a conditional approach to analyze multivariate extreme value data, proposed in Heffernan and Tawn (2004), and the key concepts involved in the model.

## 2. Theory

This section covers some of the recent research results developed from the original Heffernan and Tawn model, with regard to the the following two issues, which were previously unsolved

- Self-consistency requires the joint tail density being identical regardless of the conditioning margin

$$\mathbb{P}\left(Y_j = y_j | Y_i = y_i\right) f_{Y_i}(y_i) = \mathbb{P}\left(Y_i = y_i | Y_j = y_j\right) f_{Y_j}(y_j)$$

[13]Research partially supported by Man Investments.

- The residuals of the standardized extreme observations converge to a non-degenerate yet unidentified distribution

So this section summarizes the potential problems that these issues are associated with and proposes a feasible resolution.

# 3. Application

Motivated by some initial findings when applying the Heffernan and Tawn model to the real world financial data, this section introduces an intuitive multivariate extreme value model that accounts for the classes of assets in a typical investment portfolio. The main purpose of this section is to

- Provide some evidence to support a proposed factor-based hierarchical approach
- Describe the overall model formulation
- Illustrate its main application in cross-asset portfolio construction
- Discuss other potential applications in risk management

# 4. Conclusion

This section summarizes the main findings to date and suggests areas of future work.

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# MODELLING MORTALITY RISK WITH EXTREME VALUE THEORY

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**Abstract:** Financial losses due to unanticipated movements of mortality rates (either catastrophic mortality risk or longevity risk) have become a major concern for pension annuities. To transfer these risks to the capital market, a new risk management tool - mortality-linked securities - has been developed. Proper mortality models are therefore required to value the securities. To capture rare extreme events that cause either mortality or longevity risk, I propose a mixed model for the mortality factor $\kappa_t$. Small random fluctuations of $\kappa_t$ are modelled with a random walk with drift, while the generalized Pareto distribution in the left and/or right tail is used to incorporate extreme events. The advantage of this procedure is its unified modelling framework and its ability to extrapolate more extreme out-of-sample mortality events. Furthermore, it integrates both types of mortality risk.

## 1. Mortality Risk

Short-term, catastrophic mortality risk is the risk that over short periods of time mortality rates are much higher than would normally be experienced. Longevity risk is the uncertainty in the long-term trend in mortality rates and its impact on the long-term probability of survival of an individual. [1] These risks of unanticipated movements of the mortality rates have proved to be of greatest significance at higher ages and have lead to losses on life annuity business of life offices and pension plans. As a new risk management tool, mortality securitization enhances the capacity of the life insurance industry by transferring its catastrophic losses to financial markets. [4] The first pure mortality-risk-linked deal was the three-year Swiss Re bond issued in December 2003. Since than, several mortality-risk bonds have been issued, but no successful longevity-risk bond appeared. [2]

For the valuation of mortality-linked securities forecasts of mortality rates are needed. A suitable model, also considering cohort effects, is suggested by

Renshaw and Haberman. [6] This model postulates that the mortality rate at time $t$ for age $x$, ${}_tm_x$, satisfies the equation:

$$\ln({}_tm_x) = \alpha_x + \beta_x^{(1)}\kappa_t + \beta_x^{(2)}\gamma_{t-x}.$$

$\alpha_x$ is the age pattern of death rates, $\kappa_t$ the time-varying mortality factor, $\gamma_{t-x}$ is a random cohort effect that is a function of the year of birth $(t-x)$, and $\beta_x^{(1)}$ and $\beta_x^{(2)}$ are age-specific reactions. For the forecasts of mortality rates the mortality factor $\kappa_t$ has to be further specified.

# 2. Mortality Rates with Extreme Value Theory

A challenge in mortality risk modelling is the knowledge about rare mortality events, as there are only a few episodes of extreme mortality improvement or deterioration. The statistical methods for evaluating extreme events require an accurate measure of the tail of the distribution. Extreme value theory (EVT) can be used not only to model the given sample of observations in the tail, but also to extrapolate the probability of even more extreme, out-of-sample events. EVT can be applied to maxima or exceedences. I am interested in the exceedences in the tail of the distribution, because the payoff of the mortality indexed bond occurs when mortality exceeds or falls below a certain level. In this study I specified $\kappa_t$ using a mixed model in order to distinguish small random fluctuations of the mortality factor from jumps. Thus, this model captures explicitly the extreme mortality risk (either catastrophic mortality risk or longevity risk) as I used a random walk with drift for the small variations in the center of the distribution and the generalized Pareto distribution (GPD) above a threshold.

Let $\mathscr{X}_t = \hat{\kappa}_t - \hat{\kappa}_{t-1}$, where $\hat{\kappa}_t$ are the estimates of the Renshaw and Haberman model. $\mathscr{X}_t$ can be interpret as the mortality improvement in year $t$. In the following three general cases can be distinguished: case 1 incorporates only mortality risk, case 2 incorporates mortality and longevity risk, and case 3 incorporates only longevity risk. For case 1 only the upper tail of the distribution and for case 3 only the lower tail of the distribution follow a GPD. In case 2 both tails are modelled via a GPD, while the center is a truncated normal distribution. To estimate the density of an unimodal, possibly asymmetric random variable $\mathscr{X}$ with cdf $F$, I applied a computer intensive maximum likelihood procedure to fit the model

$$F(\chi) = p_l G^l_{\varsigma_l,\xi_l}(\chi - t_l) + (1 - p_l - p_r) N^{t_r}_{t_l}(\chi - t_r) + p_r G_{\varsigma_r,\xi_r}(\chi),$$

where $p_l$ and $p_r$ are the proportions of data in the left and right tails, $G^l_{\varsigma_l,\xi_l}$ and $G_{\varsigma_r,\xi_r}$ are the GPD models corresponding to the left and right tails,

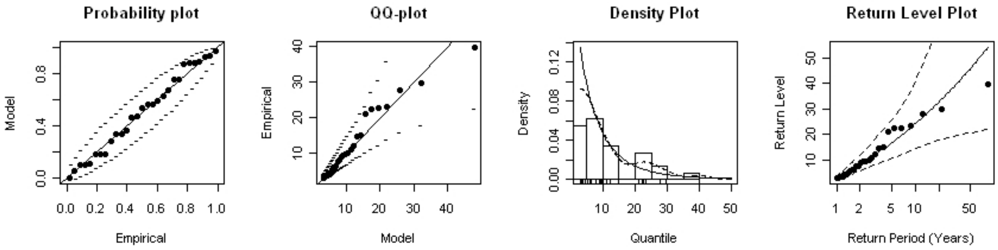


Figure 2.9: Diagnostic plots for GPD to the change of the mortality factor for France for case 2

$G^l_{\varsigma_l,\xi_l}(\chi) = 1 - G_{\varsigma_r,\xi_r}(\chi)$, and $N^{t_r}_{t_l}$ the standard normal distribution truncated at $t_l$ and $t_r$, $t_l < 0$ and $t_r > 0$. [5] For case 1 $p_l$ is zero and no left truncation exists. For case 3 $p_r$ is zero and no right truncation exists. With this general model mortality risk and/or longevity risk can be modelled as it explicitly incorporates extreme events. The GPD parameters are estimated using the L-moments estimation procedure. [3] The framework has been used to analyse the data of Finland, France, Sweden, and USA. The various diagnostic plots for assesing the accuracy of the GPD model are shown in Figure 2.9. None of the plots give cause to doubt the validity of the fitted model in the tails.

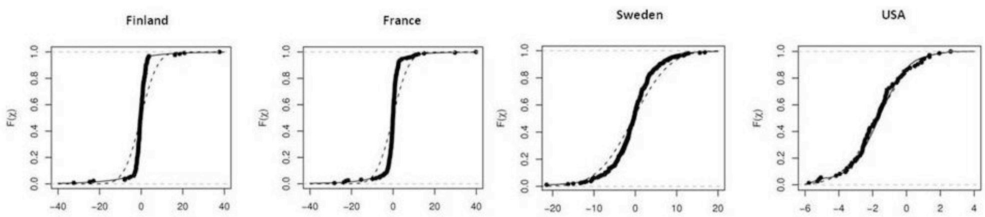


Figure 2.10: Empirical (dots), theoretical normal (dashed line), and theoretical mixed (solid line) cdf of mortality improvements $\chi$

Figure 2.10 shows the empirical cdf, the theoretical normal and theoretical mixed distribution function for case 2. In both tails the fit is improved compared with the normal distribution. The probability of large decreases or increase in mortality would be underestimated should one simply use a normal distribution to model the data. The estimator of the mixed distribution can be used to forecast the mortality factor and hence the mortality rates.

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# CONDITIONAL EXTREME-VALUE INDEX FOR HIGH-FREQUENCY ECONOMETRICS

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**Abstract:** Estimation of the extreme-value index of the limiting distribution of maxima of stationary time-series is a well-studied issue in extreme-value literature. Data are generally assumed to be equally spaced, and temporal dependencies are often captured by ARMA or GARCH models, where, for instance, daily log returns $Y_i = \ln p_i - \ln p_{i-1}$ are considered. For the case of irregularly spaced data (e.g., high-frequency transaction data), this work intends to provide a notion of "conditional extreme-value index", where "conditional" means that the local tail behavior at some point $t$ may depend on the pattern of transaction times around that point. The implementation of the concept is based on a marked point process (MPP) interpretation of irregularly spaced data and on the approach of regular variation in the derivation of the Hill estimator. An application to stock exchange transaction data yields that thickness of tails increases with trading intensity and proximity to other transactions, which is in line with known results from volatility modeling. Astonishingly, we also observe an asymmetry between backward and forward time intervals. Finally, the estimation

procedure is validated with simulated data.

# 1. Introduction

Modeling the unobserved (stochastic) volatility of an asset is an ongoing topic in econometric research. It is well-known that, for example, the underlying volatility generally increases in times of high trading intensity.

While volatility reflects the variance of the return distribution, i.e. an average deviation from the mean, the tail behavior can provide a more profound insight into the riskiness of an asset. Naturally, the question arises whether results and techniques from volatility analysis can be transferred to the tails of the return distribution.

One approach dealing with a time-varying tail behavior in consequence of instationarity is given in [2]. Therein, the rescaled maximum $\{Y(t)\}_{t\in[0,1]}$ of iid processes is allowed to have different tail indices for different locations: $\mathbb{P}(Y(t) \leq x) = \exp(-(1+\xi(t)x)^{-1/\xi(t)}) =: \mathrm{GEV}(x,\xi(t))$ and $\xi(\cdot)$ is called the *index function*. In our context of transaction data, we rather ask for an interaction between the trading intensity or the pattern of observed transaction times and the local tail behavior. The latter can be measured in terms of the tail index, for instance.

It is often difficult to distinguish between the scale and the shape parameter when estimating GEV parameters. Nonetheless, our work focuses solely on the latter one, because $\xi$ naturally allows for a moment-based definition applicable to irregularly spaced data.

# 2. Definition of the conditional tail index

Following a standard procedure in high-frequency econometrics, we perceive transaction data—consisting of time-stamps and inter-transaction log returns—as realizations of a stationary ergodic marked point process $\Phi = \{(t_i, y_i)\}_{i\in\mathbb{N}} = \{(t_i, y(t_i))\}_{i\in\mathbb{N}}$. (Here, $y(t)$ simply denotes the mark at a point $t \in \Phi$.) Let $\Phi_{\mathrm{u}}$ denote the corresponding unmarked process, i.e. $\Phi_{\mathrm{u}} = \{t_i : (t_i, y_i) \in \Phi\}$.

[3] introduces the bidirectional E- and and V-function representing the conditional mean and variance of a mark at time $t$, respectively, given that there exists another observation at time $t+r$, say. We now transfer the idea of conditional moments to extreme-value indices via a moment-based representation of the GEV shape parameter $\xi$: Assuming that the marginal distribution $F$ of a

log return $Y$ belongs to the MDA of the Fréchet distribution $\Phi_\alpha$, we have that $\mathbb{E}[\log Y - u | \log Y > u] \to \xi = \alpha^{-1}$ for $u \to \infty$ (e.g., equation (6.31) in [1]).

An intuitive definition of the point process analogon is hence given by

$$\xi(I) = \lim_{u\to\infty} \xi(I,u) \text{ with}$$
$$\xi(I,u) = \mathbb{E}\left[\log y(t) - u \,\middle|\, t \in \Phi_u, \log y(t) > u, \exists s \in \Phi_u : s - t \in I\right]. \tag{2.8}$$

Here, the interval $I$ measures the distance to the second point we condition on. To define $\xi(r,u)$ for a fixed distance $r \in \mathbb{R}$ in a mathematically precise way, we reformulate (2.8) in terms of Radon-Nikodym derivatives. In case that $\Phi$ is non-ergodic, the probability measure induced by $\Phi$ can be decomposed into a mixture of ergodic measures and the appropriate definition of the conditional tail index is then also a mixture based on (2.8).

# 3. Application

Estimation of $\xi(I,u)$ and $\xi(r,u)$ basically relies on the ergodicity assumption, which allows us to replace the expectation in (2.8) by temporal averages over an increasing observation window of one single realization.

Figure 2.11 gives an impression of the typical appearance of an estimation of $\xi(r,u)$. In this particular example, about 950.000 transactions from 2004 belonging to one single German stock index company have been used for estimation. Only the lower tail is considered (i.e. $y(t) = -(\log \text{return})(t)$).

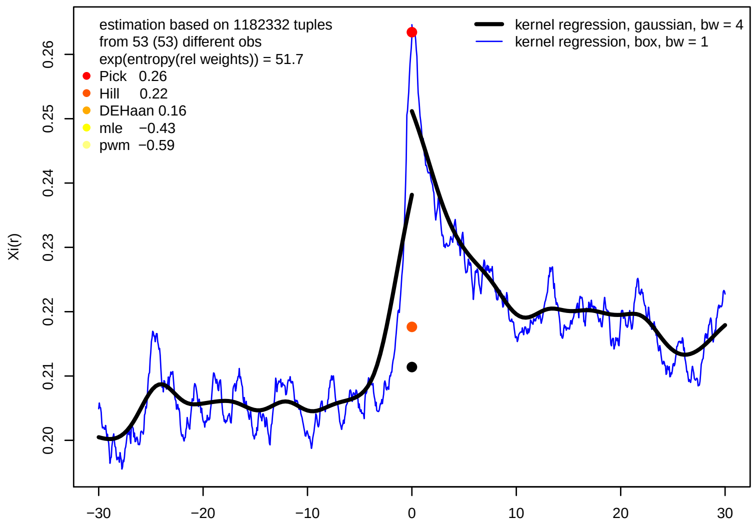


Figure 2.11: $\hat{\xi}(r,u)$ for $u$ the 99.61%-quantile, $r \in [-30,+30]$ minutes.

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# RISK FUNCTIONALS USING DISCONTINUOUS DISTORTION FUNCTIONS[14]

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**Abstract:** We introduce a family of functionals which differ from usual Choquet integrals of distorted probability measures only when the distortion function is discontinuous and give some representations of these new functionals.

## 1. Distorted probabilities and motivation

In order to measure the risk of a given portfolio modelled by a random variable $X$ risk functionals are employed with the purpose of calculating a number which represents a capital requirement that must be met so that $X$ is considered acceptable (see Föllmer and Schied (2004) for more details on the important classes of coherent and convex risk measures). Here, we introduce a new class of functionals by assuming discontinuous distortion functions and compare it with Choquet integrals as it is explained immediately.

Given a reference probability space $(\Omega, \mathscr{F}, P)$ and let $\psi : [0,1] \to [0,1]$ be an increasing function such that $\psi(0) = 0$ and $\psi(1) = 1$. The *distorted probability* generated by $\psi$ is given by $P^* := \psi \circ P$ where $\psi$ takes the name of *distortion function.* The expectation of random variables $X$ under the distorted probability $P^*$ is given by

$$\int X\, dP^* := \int_{-\infty}^{0} \left(P^*(X > x) - 1\right) dx + \int_{0}^{\infty} P^*(X > x)\, dx$$

which is a particular case of the more general *Choquet integrals*.

The main motivation for our work comes from the following result presented in Föllmer and Schied (2004). There it is proved that if $\psi$ is concave

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and continuous at the origin then

$$\int (-X)\, dP^* = -\int_0^1 q_X(\varphi(s))\, ds\,. \tag{2.9}$$

where $\varphi : [0,1] \to [0,1]$ is any pseudo-inverse of $\psi$ and $q_X$ is the pseudo-inverse (or in standard nomenclature, *quantile function*) of the distribution function $F_X := P(X \leq x)$.

Our goal is then to look for possible generalizations of (2.9) under the more general assumption that $\psi$ has discontinuities. For the purpose of clarity we name integrals appearing in the right-hand side of (2.9) by *integrals of distorted quantile functions*.

# 2. Quantile functions intertwined

In addition to the distribution function $F_X$ let us also define a distorted version $F_X^* := P^*(X \leq x)$; their quantile functions will be denoted by $q_X$ and $q_X^*$, respectively; the *decreasing distribution functions* are defined as being, $G_X(x) := P(X > x)$ and $G_X^*(x) := P^*(X > x)$, and their quantile functions are denoted by $r_X$ and $r_X^*$, respectively. Lower and upper bounds of any quantile functions will be represented by the standard notation, *e.g.*, $r_X^-$ and $r_X^+$, respectively. We then have the following identities:

$(i)$ if $\psi$ is right-continuous then for all $s \in (0,1)$ we have that

$$(q_X^*)^-(s) = q_X^-\left(\varphi^-(s)\right) \text{ and } (r_X^*)^+(s) = q_X^+\left(1-\varphi^-(s)\right);$$

$(ii)$ if $\psi$ is left-continuous then for all $s \in (0,1)$ we have that

$$(q_X^*)^+(s) = q_X^+\left(\varphi^+(s)\right) \text{ and } (r_X^*)^-(s) = q_X^-\left(1-\varphi^+(s)\right).$$

# 3. Integrals of distorted quantile functions

In this section we obtain representations for integrals of the form

$$\int_0^t q_X(\varphi(s))\, ds\,.$$

## 3.1. General case: $0 \leq t \leq 1$

Let $\kappa_X$ denote a quantile function of $q_X \circ \varphi$. Then it follows that

$$\int_0^t q_X(\varphi(s))\, ds = \int_0^\infty (t - \kappa_X(x))\, dx - \int_{-\infty}^0 \kappa_X(x)\, dx$$

and, moreover

$$\psi^{+}(P(X \leq x)) \geq \kappa_X^{-}(x) \text{ and } \psi^{-}(P(X \leq x)) \leq \kappa_X^{+}(x)$$

which implies that if $\psi$ is continuous then a possible choice for the quantile function is $\kappa_X(x) = \psi(P(X \leq x))$ for all $x \leq q_X(\varphi(t))$.

This yields a representation of integrals of distorted quantile functions similar to that of Choquet integrals which recovers identity (2.9) in the case when $\psi$ is continuous and $t = 1$.

Also, from a change of variables (see Ignacio and Mendes (2010)) we have that

$$\int_0^t q_X(\varphi(s))\, ds = \int_0^u q_X(x)\,\psi'(x)\, dx + \\ + \sum_{x\in[0,u)} q_X(x)\left\{\psi\left(x^{+}\right) - \psi\left(x^{-}\right)\right\} + q_X(u)\left\{t - \psi\left(u^{-}\right)\right\}$$

where $u = \varphi(t)$. Note that $\sum_{x\in[0,u)} q_X(x)\{\psi(x^{+}) - \psi(x^{-})\} < \infty$ when $X$ is bounded since $\sum_{x\in[0,u)}\{\psi(x^{+}) - \psi(x^{-})\} \leq 1$.

This last equality provides a decomposition of our functionals in terms of *spectral functionals* (see Acerbi (2002)) and quantile functionals which are known as VaR measures in the financial context.

## 3.2. Special case: $t = 1$

A simple application of the relations between quantile functions presented in Section 2 yields the following results.

$(i)$ If $\psi$ is right-continuous then

$$\int X\, dP^{*} = \int_0^1 q_X^{+}(1 - \varphi(s))\, ds \text{ and } \int (-X)\, dP^{*} = -\int_0^1 q_X^{-}(\varphi(s))\, ds\,.$$

$(ii)$ If $\psi$ is left-continuous then

$$\int X\, dP^{*} = \int_0^1 q_X^{-}(1 - \varphi(s))\, ds \text{ and } \int (-X)\, dP^{*} = -\int_0^1 q_X^{+}(\varphi(s))\, ds.$$

Again, for continuous $\psi$ we recover formula (2.9).

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# A Heuristic Data-driven Choice of Tuning Parameters in PORT-MVRB Estimation[15]

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**Abstract:** On the basis of second-order *minimum-variance reduced-bias* (MVRB) *extreme value index* (EVI) estimators, based on $k$ top order statistics, we consider *peaks over random threshold* (PORT)-MVRB EVI–estimators, dependent on an extra *tuning* parameter $q$. Based on bias patterns, we propose a heuristic algorithm for the adaptive choice of $k$ and $q$, and apply it in the fields of insurance and finance.

## 1. Introduction

For heavy-tailed parents, given a sample $\underline{\mathbf{X}}_n = (X_1, \ldots, X_n)$ and the associated sample of ascending order statistics (o.s.'s), $(X_{1:n} \le \cdots \le X_{n:n})$, the classical EVI-estimators are Hill estimators ([7]), denoted $H \equiv H(k) \equiv H(k; \underline{\mathbf{X}}_n)$, the average of the $k$ log-excesses over a high random threshold $X_{n-k:n}$, $1 \le k < n$. These estimators are scale-invariant, but not location-invariant, and this contrarily to the PORT-Hill estimators, introduced in [1] and further studied in [5]. The PORT-Hill estimators are based on

$$\underline{\mathbf{X}}_n^{(q)} := \left(X_{n:n} - X_{[nq]+1:n}, \ldots, X_{[nq]+2:n} - X_{[nq]+1:n}\right),$$

a sample of excesses over a random threshold $X_{[nq]+1:n}$, with $[x]$ denoting the integer part of $x$. We can generally have $0 < q < 1$, for d.f.'s with finite or

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infinite left endpoint $x_F := \inf\{x : F(x) > 0\}$ (*the random threshold is an empirical quantile*), and $q = 0$, for d.f.'s with finite left endpoint $x_F$ (*the random threshold is the minimum*). They shall be denoted $H^{(q)} \equiv H^{(q)}(k) := H(k; \underline{\mathbf{X}}_n^{(q)})$, $1 \le k < n - [nq] - 1$, i.e., they have the same functional form of the Hill estimator, but with $\underline{\mathbf{X}}_n$ replaced by $\underline{\mathbf{X}}_n^{(q)}$. All these EVI-estimators reveal usually a high asymptotic bias. Hence the need for bias correction. A simple class of second-order MVRB EVI-estimators is the one in [3]. This class, denoted $\overline{H} \equiv \overline{H}(k)$, depends upon the estimation of a vector $(\beta, \rho)$ of second-order parameters, and it is given by

$$\overline{H}(k) \equiv \overline{H}(k; \underline{\mathbf{X}}_n) := H(k)\big(1 - \hat{\beta}(n/k)^{\hat{\rho}}/(1-\hat{\rho})\big).$$

We shall now consider the EVI-estimators $\overline{H}^{(q)}(k) := \overline{H}(k; \underline{\mathbf{X}}_n^{(q)})$, the so-called PORT-MVRB estimators, invariant for both changes of location and scale. Section 2 is dedicated to a data-driven choice of the *tuning parameters k* and $q$, inspired in the heuristic choices considered in [4] for the Value-at-Risk estimation and in [6] for the EVI-estimation, and based on the reasonably high stability on $k$ of the MVRB estimates $\overline{H}$, and $\overline{H}^{(q)}$, for adequate $q$. Finally, in Section 2, we provide applications of the adaptive methodology to data in the fields of insurance and finance, as well as to a simulated sample from a Student underlying parent.

# 2. Data-driven choices and data analysis

With the notation $X_{n+1:n} = 0$, we can consider that, by convention, $\overline{H} = \overline{H}^{(q)}$ for $q = 1$, and work with $q \in [0, 1]$. Our interest lies then on the estimation of $\gamma$ through $\overline{H}^{(q)}$, also including $\overline{H}$. Based on the stability on $k$ of the MVRB estimates, and of the PORT-MVRB estimates for adequate values of $q$, we propose the following method for an adaptive heuristic estimation of $\gamma$.

## 2.1. An algorithm for the heuristic choice of $k$ and $q$

1. Given an observed sample $(x_1, x_2, \ldots, x_n)$, consider, for $q = 0(0.1)0.5, 1$, the observed sample of excesses, $\underline{\mathbf{x}}_n^{(q)}$.

2. Next compute, for $k = 1, 2, \ldots, n - [nq] - 1$, the observed values of $\overline{H}^{(q)}(k)$ $\big($note that, as mentioned before and by convention, $\overline{H}^{(1)} \equiv \overline{H}\big)$.

**3.** Consider the smallest number of decimal places that enables variation in the observed values of $\overline{H}^{(q)}(k)$. For each $q$ consider as estimates of $\gamma$ the values $\overline{H}^{(q)}(k)$, $k_{min}^{(q)} \le k \le k_{max}^{(q)}$, to which is associated the largest run, with size $l_q = k_{max}^{(q)} - k_{min}^{(q)} + 1$. Choose $q_0 := \arg\max_q l_q$;

**4.** Consider the estimates $\overline{H}^{(q_0)}(k)$, $k_{min}^{(q_0)} \le k \le k_{max}^{(q_0)}$, with an extra decimal place. Count their frequencies and obtain the mode, $m_0$, of these values. Let us denote $\mathscr{K}$ the associated set of $k$-values. Take $k_0$ as the maximum of $\mathscr{K}$, and the adaptive EVI-estimate $\hat{\gamma} = \overline{H}_0^{(q_0)}(k_0)$.

## 2.2.1. Case studies and a simulated data set

We shall consider an illustration of the performance of the *Algorithm*, when applied to the analysis of

1) automobile claim amounts over the period 1988–2001(Secura Belgian Re), studied in [2], among others;
2) the log-returns associated with the daily closing values of the Microsoft Corp. (MSFT);
3) an arbitrarily simulated random sample of size $n = 1762$, from a Student's $t_\nu$-model with $\nu = 4$ degrees of freedom.

In Figure 2.12, we present the estimates of $\gamma$, provided by $H$ and $\overline{H}^{(q)}$, $q = 0, 0.1, 0.2$ and 1 ($\overline{H} \equiv \overline{H}^{(1)}$) for the above-mentioned data sets.

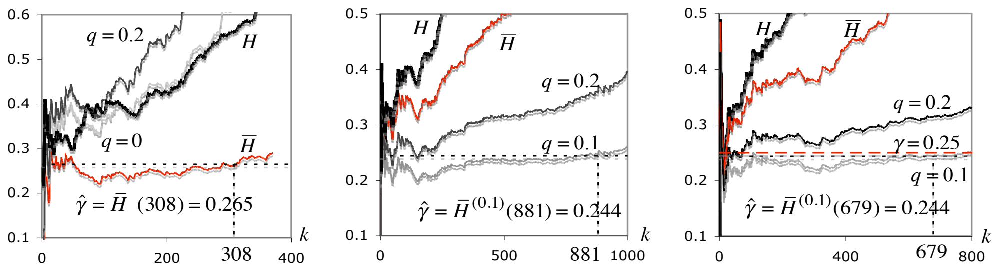


Figure 2.12: EVI-estimates, as a function of $k$, for the SECURA (*left*) the MSFT (*center*) and a Student $t_4$ samples (*right*), as well as the adaptive estimates obtained through the *Algorithm*.

- The model underlying SECURA data set has a finite positive left endpoint. We thus expect to have no improvement through the use of the PORT methodology, and we were indeed led to the choice of the MVRB EVI-estimator, $\overline{H}$.

- These case studies claim for a simulation study of the *Algorithm* presented in Section 2, a topic out of the scope of this paper. However, the results obtained clearly indicate an over-estimation of the adaptive Hill estimate and an overall best performance of this data-driven (and location-invariant) method of EVI-estimation.

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# THE CROSS-SECTION OF TAIL RISKS IN STOCK RETURNS

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## 1. Abstract

Downside risks of financial assets is of a major concern for investors and risk managers, witness the popularity of Value-at-Risk (VaR) constraints in the banking industry. The unfolding of the recent financial crisis further raises the necessity of assessing the tail risks of potential extreme losses, often referred to as *Black Swan* events in the popular vernacular. This paper analyzes the tail risks in the equity market at a firm level and further identifies cross-sectional drivers for the tail risks from firm accounting data. Identifying the driving factors at a firm level help investors evaluate potential tail risks when holding equities from different firms. Based on the evaluation of firm level accounting data, investors can construct a portfolio of equities which accommodates both their desired return and downside risk appetite. From a corporate finance viewpoint, with understanding of the drivers of tail risks, firms can steer their

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management towards a lower tail risk taking. This can facilitate and enhance access to capital markets by attracting long-term risk adverse investors.

Recent empirical literature addresses that modeling losses from financial assets, particularly equities, with Gaussian distributions underestimate the probability of extreme negative returns. Instead, there is large consensus in the literature that the distribution of asset returns is heavy-tailed. Under this assumption, large negative equity returns can be modeled using Extreme Value Theory (EVT). Under a heavy-tailed setup the VaR measure can be easily assessed by estimating the shape and scale of the tail distribution.

In this paper, we follow the heavy-tailed setup on equity returns and attempt to find potential drivers from firm level accounting information that are associated with the tail risks measured by the VaR. A comparable work has been done by Wu, Huang, Liu, and Rhee (2010). Following the heavy-tailed setup, they use the tail index, which indicates the shape of the tail distribution, as the measure of tail risk and investigate potential covariates at a firm level. However, Hyung and de Vries (2002) find evidence that the shape of the tail distribution does not vary across firms that have equities traded on the same market. If this is indeed the case, then using the shape of the tail distribution, or the tail index, may not provide an adequate measure for differentiating between firm level tail risks.

Following the Capital Asset Pricing Model (CAPM), we build models on the tail risks of equity returns and provide theoretical proof that the tail indexes should remain at a constant level across different firms from the same market. We show that the inequality of the tail indices do not allow for benefits from portfolio diversification and this leads to arbitrage opportunities. Under the no-arbitrage hypothesis, we conclude that the tail indices are equal. This motivates us to focus on the secondary parameter in VaR calculation, the scale. The model shows that the scale of the tail distribution of equity returns should vary across different firms according to their risk attitudes. An empirical study on 291 US firms supports the theoretical findings. With a formal statistical test, we find no evidence that the shape parameter, that is, the tail index, differs across equities in our data set. However, applying a similar methodology to the scale shows that this parameter varies dramatically cross-sectionally across equities. We show that this result leads to benefits of portfolio diversification.

In order to explain the differences in the scale parameter, and therefore the tail risk of equity returns, we find firm level determinants that drive the cross-sectional differences. A cross-sectional regression of the scale parameter against a vector of firm-level balance sheet data reveals that the market capitalization, earnings-price ratio, and the share turnover rate are all significant in explaining these differences, and thus driving tail risk. Other potential factors

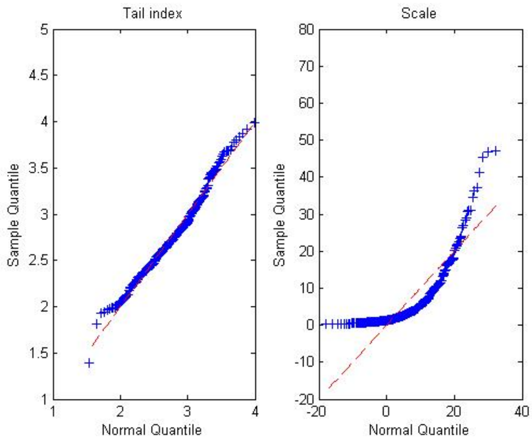


Figure 2.13: Normal QQ-plots demonstrating equality and inequality.

such as book-to-market ratio, debt-equity ratio, and the bid-ask spread do not seem to significantly contribute to the driving of tail risk.

Table 2.4: Regression of Scale against firm-level characteristics

| | 2000-03 | 01-04 | 02-05 | 03-06 | 04-07 | 05-08 |
|---|---|---|---|---|---|---|
| SIZE | -0.37** | -0.35** | -0.36** | -0.40** | -0.38** | -0.24** |
| GROWTH | 0.03 | -0.00 | -0.05 | -0.15** | -0.16** | -0.02 |
| LEVERAGE | -0.03 | -0.05 | -0.03 | -0.02 | -0.00 | 0.00 |
| YIELD | -1.28** | -3.33** | -3.50** | -4.49** | -1.85** | -0.26** |
| TURNOVER | 0.75** | 0.52** | 0.51** | 0.58** | 0.56** | 0.37** |
| SPREAD | 0.35* | 0.38* | 0.36* | 0.27 | 0.20 | 0.07 |

** 1%, and * 5% significance respectively

While some of these results are in agreement with the prevailing literature concerning risk firm-level risk factors, some of the results are also conflicting. This motivates us to consider the idea of assets that appear less risky during “normal” market conditions may, in fact, pose high risks to investors during adverse periods in the market giving rise to portfolio diversification considerations for investors over all market contingencies. Our results also give reason to believe that firm size, earnings yield, and share turnover are ”non-diversifiable” factors that contribute to the tail risk of equity returns.

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# EXTREMAL INDEX ESTIMATION THROUGH AN ADAPTIVE RESAMPLING APPROACH[18]

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**Abstract:** The *extremal index*, $\theta$, is a key parameter in extreme value theory for initial underlying dependent samples, measuring the degree of local dependence in the largest observations of a stationary process. Its estimation is also important because of its effect on the estimation of other parameters of extreme events. The estimators considered in the literature, despite of having good asymptotic properties, show a strong dependence on the high level $u_n$, presenting a high variance for high levels and a high bias when the level decreases. A compromise between bias and variance is obtained by considering the mean squared error, $MSE$. A question often addressed is the choice of $u_n$ that minimizes $MSE$. An adaptive resampling approach is here considered for estimating the level $u_n$ that asymptotically gives the minimum $MSE$ of an estimator of the extremal index. Simulation studies as well as real cases have been performed. Here an application to daily returns of the S&P 500 stock index is presented.

# 1. Introduction

The *extremal index* is an important parameter measuring the degree of clustering of extremes in a stationary process. There exist several interpretations of the *extremal index* from which several estimators have been derived. One of those interpretations, due to Leadbetter (1983), considers $\theta$ as the reciprocal of the limiting mean cluster size. The identification of clusters of high level exceedances is then required.

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The most basic form of cluster identification was to consider that a cluster occurred whenever there was an up-crossings of the high threshold $u_n$. This suggested the *up-crossing estimator*, $\widehat{\theta}_n^{UC}$, (Leadbetter and Nandagopalan,1989 and Gomes, 1990), defined for a random sample $(X_1, X_2, \ldots, X_n)$ and a suitable threshold $u_n$ as:

$$\widehat{\theta}_n^{UC} := \frac{\sum_{i=1}^{n-1} I(X_i \leq u_n < X_{i+1})}{\sum_{i=1}^{n} I(X_i > u_n)}.$$

This estimator does not require any knowledge of clustering characteristics of the process. However, like other estimators it is strongly dependent on the high level $u_n$ showing a high variance for high levels and a high bias when the level decreases. Regarding the compromise between these two measures given by the *mean squared error*, *MSE*, a resampling scheme and an adaptive procedure is performed for estimating the optimal level $u_n$ (that that minimizes *MSE*) and then for obtaining an estimate of $\theta$.

# 2. Resampling in stationary processes and the optimal sample fraction estimation

Bootstrap methodology, Efron(1979), was first introduced in the context of i.i.d. data, as an auxiliary estimation procedure for providing answers to many complex problems. However, Singh (1981) showed the inadequacy of the classic bootstrap in the context of dependent data. His idea was to group the observations into blocks and to carry out the resampling at the block level. The motivation for this scheme is that the dependence structure of the underlying model is preserved within each block. Several authors studied ways of blocking: the nonoverlapping block bootstrap, the moving block bootstrap, the circular block bootstrap and the stationary bootstrap.

The performance of a block resampling method critically depends on the particular block length employed in finding the bootstrap estimator. Hall *et al.* (1995) proposed a subsampling method to define a data-based version of *MSE* function which is minimized and rescaled to produce an estimator of the optimal block size.

Lahiri *et al.* (2003) proposed a plug-in rule as an alternative approach for empirical choice of the optimal block size. The key idea of the method is based on the bootstrap estimation of the variance and the bias of the block bootstrap estimator. The proposed rule is based on the Jackknife-After-Bootstrap (JAB) that yields a nonparametric estimator of the variance of a block bootstrap estimator, briefly described below, for the moving block bootstrap case, that was considered in this work.

Given the observations, $\underline{X}_n = (X_1, X_2, \ldots, X_n)$, let $\widehat{\theta}_n^*(b)$ be the moving block bootstrap estimator of $\theta$ based on blocks of length $b$ and let $\ell = n-b+1$ be the number of "observable" blocks of length $b$.

Let $m \equiv m_n$ be an integer such that as $n \to \infty$, $m \to \infty$ and $m/n \to 0$, denoting the number of bootstrap blocks to be deleted. Write $M = \ell - m + 1$ and for $i = 1, \cdots, M$ let us define the set $I_i = \{1, \cdots, \ell\} \backslash \{i, \cdots, i+m-1\}$ denoting the index set of all blocks obtained by deleting the $m$ blocks. Let us denote by $\{B_j : j \in I_i\}$ the reduced collection of blocks. We resample $[n/b]$ blocks from $\{B_j : j \in I_i\}$ and compute the j*th* jackknife block deleted point value $\widehat{\theta}_n^{(i)*}(b)$ for $i = 1, \cdots, M$. The JAB estimator for the variance of $\widehat{\theta}_n^*(b)$ is defined as:

$$\widehat{VAR}_{JAB}[\widehat{\theta}_n^*(b)] = \frac{m}{(\ell-m)M} \sum_{i=1}^{M} \left[ \widetilde{\theta}_n^{(i)*}(b) - \widehat{\theta}_n^*(b) \right]^2$$

where $\widetilde{\theta}_n^{(i)*}(b) = m^{-1} \left[ \ell \widehat{\theta}_n^*(b) - (\ell - m) \widehat{\theta}_n^{(i)*}(b) \right]$ is the i*th* block-deleted jackknife *pseudo-value* of $\widehat{\theta}_n^*(b)$, $i = 1, \cdots, M$. The authors proposed a new estimator for the bias of the block bootstrap estimator, defined as:

$$\widehat{BIAS}[\widehat{\theta}_n^*(b)] = 2 \left[ \widehat{\theta}_n^*(b) - \widehat{\theta}_n^*(2b) \right].$$

The nonparametric plug-in estimator of the optimal block length is given by:

$$\widehat{b}_{opt} = \left[ \frac{2\widehat{C}_2^2}{r\widehat{C}_1} \right]^{\frac{1}{r+2}} n^{\frac{1}{r+2}},$$

where $\widehat{C}_2 = 2b \left\{ \widehat{\theta}_n^*(b) - \widehat{\theta}_n^*(2b) \right\}$ and $\widehat{C}_1 = (nb^{-r}) \widehat{VAR}_{JAB}[\widehat{\theta}_n^*(b)]$.

According to some suggestions given in Lahiri *et al.* (2003) for obtaining $\widehat{C}_1$ and $\widehat{C}_2$, as a first approach we used $b = c_1 n^{1/(r+4)}$, with $c_1 = 1$ and $r = 1$ and with this choice of $b$, $m = c_2 n^{1/3} b^{2/3}$ with $c_2 = 1$.

Once obtained an estimate for the optimal block length, an adaptive procedure is carried out and the optimal sample fraction is obtained and finally $\theta$ is estimated.

An extensive simulation study is in progress and an application to daily returns of the S&P 500 stock index is presented.

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# WITHIN-CLUSTER BEHAVIOUR OF THE EXTREMES OF STOCK MARKET INDICES[19]

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**Abstract:** To characterize the extremal behaviour of stationary time series, attention has been given to the within-cluster behaviour of the extremes of a series, which is determined by the short-range temporal dependence. Most of its characterization has been done based on the assumption of Markovianity of the time series, as the class of $d$th-order Markov chains is sufficiently general and tractable. A simple quantifier of the extremal within-cluster dependence for any stationary time series satisfying the Leadbetter *et al.* (1983) $D$ condition is the threshold dependence extremal index, $\theta(u)$, which measures the extent of clustering of exceedances of the process above the high threshold $u$. In this work, we study and estimate this function for asymptotically independent Markov chains and model temporal dependence within these series extremes. Applying these techniques to daily log-returns of stock indices, we examine large consecutive changes associated with large financial gains or losses.

# 1. Introduction

To describe the behaviour of exceedances of a stationary process, attention has been given to the within-cluster behaviour of the extremes of Markov chains, which is determined by the short-range temporal dependence. The assumption of Markovianity of the time series is based on the class of $d$th-order Markov chains being sufficiently general and tractable. The simplest limiting quantifier of the extremal within-cluster dependence for any stationary time series $\{X_n\}_{n\geq 1}$ satisfying the Leadbetter *et al.* (1983) $D$ condition

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is the extremal index $\theta$ with $0 \leq \theta \leq 1$. Values of $\theta$ close to 0 indicate a very strong short range extremal dependence, while values close to 1 suggest a rather weak dependence. In fact, $\theta$ is both the reciprocal of the limiting mean cluster size and the only parameter describing the effect of short-range extremal dependence on the limiting distribution of the componentwise maxima $M_N = \max\{X_1, \ldots, X_N\}$, in the sense that there exists a sequence of constants $a_N > 0$, $b_N$ and $u_N = a_N x + b_N$ such that

$$\lim_{N\to\infty} \Pr(M_N \leq u_N) = \lim_{N\to\infty} \{F(u_N)\}^{N\theta},$$

where $F$ is the common marginal distribution function and $\lim_{N\to\infty}\{F(u_N)\}^{N\theta} = G^{\theta}(x)$ for a non-degenerate distribution function $G$ called the generalized extreme value distribution. However, some difficulties arise in the case $\theta = 1$, which implies that asymptotically extreme events occur singly. For example, for all asymptotically independent Markov processes $\{X_n\}_{n\geq 1}$, i.e. such that $\lim_{u\to x^*} \Pr(X_{i+1} > u | X_i > u) = 0$ where $x^*$ is the upper limit of the support of the common marginal distribution, the extremal index is one, despite clustering of exceedances of high thresholds still occurring, see Bortot and Tawn (1998). For such processes it is useful to characterize the extremal dependence by a penultimate version of the extremal index, $\theta(u) \equiv \theta(u,m)$ say, defined as a function of the high threshold $u$ and $m = m(u)$ the length of the window of the clusters, and whose limit is $\theta$ as $u \to x^*$. Following Bortot and Tawn (1998), it is natural to define $\theta(u,m)$ as

$$\theta(u) \equiv \theta(u,m) = [E\{N(u,m)|N(u,m) \geq 1\}]^{-1}, \tag{2.10}$$

where $N(u,m) = \sum_{i=1}^{m} I(X_i > u)$ is the size of a cluster with window of length $m$ initialized at $i = 1$, and where $I$ is the indicator function. Provided a suitable choice of $m$ can be made, $\theta(u,m)$ can be estimated by an empirical version of (2.10).

# 2. Within-clusters behaviour at extreme levels

Having fitted an adequate Markov model to the stationary time series, estimates of the threshold dependent extremal index $\theta(u)$ can be obtained by simulation of clusters of extreme events from the fitted model. Using the dependence Markov model for the extremes of time series described in Ramos (2009), we follow the methodology in Bortot and Tawn (1998). Then, we first simulate the maximum value of the cluster and then simulate values backwards and forwards over the cluster window according to the transition density of the fitted Markov model. Then, the complete algorithm for the cluster simulation is as follows.

1. **Choice of threshold and length of cluster window.** Choose a high threshold $u$, in a unit Fréchet scale, such that $u > u_f$ and choose $m = 2r+1$, where $m$ and $r$ are the length and the radius of the window, respectively.

2. **Simulation of the cluster maximum.** Simulate the cluster maximum, $Y_0$, from a GPD.

3. **Simulation of the cluster.** Simulate the observations $Y_1, \ldots, Y_r$ iteratively from a df $H_F^*$ and simulate the observations $Y_{-1}, \ldots, Y_{-r}$ iteratively from a df $H_B^*$. Each of these sequences will be rejected whenever a value $Y_i > Y_0$ is generated, and its simulation will be repeated.

4. **Estimation of functionals of interest.** Having repeated the previous steps to produce a large number $k$ of clusters, several quantities of interest can be constructed empirically. For example, denoting by $N_j\left(u;\hat{\phi}\right)$ the number of exceedances of $u$ (of $m(u)$) in the $j$th simulated cluster that has at least one exceedance of $u$, the threshold dependent extremal index can be estimated by

$$\hat{\theta}\left(u,\hat{\phi}\right) = \left\{\frac{1}{k}\sum_{j=1}^{k} N_j\left(u;\hat{\phi}\right)\right\}^{-1},$$

where $\hat{\phi}$ represents the model estimated parameters.

# 3. Applications

These methods are applied to financial time series consisting of daily log-returns of closing prices for stock market indices. Upper and lower tails of the series are examined corresponding to large consecutive changes associated with large financial gains or losses.

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# ESTIMATING THE CONDITIONAL TAIL EXPECTATION IN THE CASE OF HEAVY-TAILED LOSSES

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**Abstract:** The conditional tail expectation (CTE) is a popular actuarial risk measure and a useful tool in financial risk assessment. Under the classical assumption that the second moment of the loss variable is finite, the asymptotic normality of the non-parametric CTE estimator has already been established in the literature. That result, however, is not applicable when the loss variable follows any distribution with infinite second moment, which is a frequent situation in practice. With a help of extreme-value methodology, in this paper we offer a solution to the problem by suggesting a new CTE estimator, which is applicable when losses have finite means but infinite variances.

# 1. Construction of a new CTE estimator

Let $X$ be a loss random variable with cumulative distribution function (cdf) $F$. Usually, the cdf $F$ is assumed to be continuous and defined on the entire real line, with negative loss interpreted as gain. We also make this continuity of $F$ assumption throughout the present paper. The CTE of the risk $X$ is then defined, for every $t \in (0,1)$, by

$$CTE(t) = E(X|X > Q(t)),$$

where $Q(t) = \inf\{x : F(x) \geq t\}$ is the quantile function corresponding to the cdf $F$. Since the cdf $F$ is continuous, we easily check that $CTE(t)$ is equal to

$$\mathbb{C}(t) = \frac{1}{1-t} \int_t^1 Q(s) ds.$$

Hence, from now on we work with $\mathbb{C}(t)$ and call it the CTE for short. Naturally, the CTE is unknown since the cdf $F$ is unknown. Hence, it is desirable to establish appropriate statistical inferential results such as confidence intervals for $\mathbb{C}(t)$ with specified confidence levels and margins of error. We shall next show how one can accomplish this task, initially assuming the classical moment assumption $E[X^2] < \infty$. Namely, suppose that we have independent random variables $X_1, \ldots, X_n$ each with the cdf $F$, and let $X_{1:n} < \cdots < X_{n:n}$ denote the order statistics of $X_1, \ldots, X_n$. It is natural to define an empirical estimator of $\mathbb{C}(t)$ by the formula

$$\widehat{\mathbb{C}}_n(t) = \frac{1}{1-t} \int_t^1 Q_n(s) ds,$$

We have already noted that the 'old' estimator $\mathbb{C}_n(t)$ does not yield asymptotic normality beyond the condition $E[X^2] < \infty$. For this reason we next construct an alternative CTE estimator, which takes into account different asymptotic properties of moderate and high quantiles in the case of heavy-tailed distributions. Hence, from how on we assume that $\gamma \in (1/2, 1)$. Before indulging ourselves into construction details, we first formulate the new CTE estimator:

$$\widetilde{\mathbb{C}}_n(t) = \frac{1}{1-t} \int_t^{1-k/n} Q_n(s) ds + \frac{k X_{n-k,n}}{n(1-t)(1-\widehat{\gamma})},$$

where we use the Hill (1975) estimator

$$\widehat{\gamma}_n = \frac{1}{k} \sum_{i=1}^{k} \log X_{n-i+1:n} - \log X_{n-k:n}$$

of the tail index $\gamma \in (1/2, 1)$. The main result are given in the following theorem.

## 2. Main theorem and its practical implementation

We start this section by noting that Hill's estimator $\widehat{\gamma}_n$ has been thoroughly studied, improved, and generalized in the literature. For example, weak consistency of $\widehat{\gamma}_n$ has been established by Mason (1982) assuming only that the underlying distribution is regularly varying at infinity. Asymptotic normality of $\widehat{\gamma}$ has been investigated under various conditions by a number of researchers, including Csörgő and Mason (1985), Beirlant and Teugels (1989), Dekkers *et al.* (1989), see also references therein.

The main theoretical result of this paper, establishes asymptotic normality of the new estimator $\widetilde{\mathbb{C}}_n(t)$. To formulate the theorem, we need to introduce an assumption that ensures the asymptotic normality of Hill's estimator $\widehat{\gamma}_n$. The assumption is equivalent to the following second-order condition (see Geluk *et al.* (1997)). Namely, the cdf $F$ satisfies the generalized second-order regular variation condition with second-order parameter $\rho \leq 0$ (see de Haan and Stadtmüller (1996)) if there exists a function $a(t)$ which does not change its sign in a neighbourhood of infinity and such that, for every $x > 0$,

$$\lim_{t\to\infty} \frac{1}{a(t)} \left( \frac{1-F(tx)}{1-F(t)} - x^{-1/\gamma} \right) = x^{-1/\gamma} \frac{x^{\rho/\gamma}-1}{\rho/\gamma}; \tag{2.11}$$

when $\rho = 0$, then the ratio on the right-hand side of equation (2.11) should be interpreted as $\log x$. For statistical inference concerning the second-order parameter $\rho$ we refer, for example, to Peng and Qi (2004), Gomes *et al.* (2005), Gomes and Pestana (2007). Furthermore, in the formulation of Theorem 2.1, we shall also use the functions $A(z) = \gamma^2 a(\mathbb{U}(z))$ and $\mathbb{U}(z) = Q(1-1/z)$; after the formulation of the theorem we shall provide illustrative examples of these functions.

**Theorem 2.1.** *Assume that the cdf $F$ satisfies condition* (2.11) *with $\gamma \in (1/2, 1)$. Then for any sequence of integers $k = k_n \to \infty$ such that $k/n \to 0$ and $k^{1/2}A(n/k) \to 0$ when $n \to \infty$, we have that*

$$\frac{\sqrt{n}\big(\widetilde{\mathbb{C}}_n(t) - \mathbb{C}(t)\big)(1-t)}{(k/n)^{1/2} X_{n-k:n}} \to_d \mathcal{N}\left(0, \sigma_\gamma^2\right)$$

*for any fixed $t \in (0,1)$, where the asymptotic variance $\sigma_\gamma^2$ is given by the formula*

$$\sigma_\gamma^2 = \frac{\gamma^4}{(1-\gamma)^4(2\gamma-1)}.$$

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# POPULATIONS WITH PERIODIC RECLASSIFICATION: APPLICATION TO THE FINANCIAL SECTOR

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**Abstract:** Our research is centred on the stochastic structure of matched open populations, subjected to periodical reclassifications. These populations are divided into sub-populations. Two or more of such population are matched when there is a 1-1 correspondence between their sub-populations and the elements of one of them can go to another, if and only if the same occurs with elements from the corresponding sub-populations of the other. It is known from literature that when the relative dimensions of the sub-populations are stable we can say that we have a stochastic vortex. The existence of such a vortex leads to the existence of a limit distribution. Matched populations may then be compared through these distributions. To obtain conditions for the existence of stochastic vortices we assumed that the entries and departures occur at the beginning of fixed length time periods; also at the beginning of those periods the new elements are allocated to the different populations and the elements in the population are reallocated; the entry and reallocation probabilities do not change from period to period. Under these assumptions the populations will have underlying homogeneous Markov chains. We intend to generalize these assumptions since they showed to be acceptable for our application. In our application we considered two populations of customers of a bank: with and without account manager. Besides these study connected with Markov chains we show how to carry

out Analysis of Variance like analysis of entries and departures to and from de populations of customers. This study was useful since it enabled as to length the model.

**Key words and phrases:** Populations with periodic reclassification, likelihood ratio tests, isomorphism, Markov chains, limit distributions, stochastic vortices.

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# ESTIMATING THE UPCROSSINGS INDEX FOR FINANCIAL TIME SERIES[20]

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**Abstract:** The upcrossings index is a measure of clustering of upcrossings of high leves by the variables of a stationary sequence and is directly related to the extremal index, for suitable high thresholds. Considering the different representations of this parameter, under general and local asymptotic dependence restrictions, we propose estimators for it. The performance of the estimators is assessed through applications to real financial time series.

## 1. Introduction

The extremal index of a stationary time series describes its tendency to cluster by observations above high thresholds. This phenomena, very common in financial time series, is referred to as volatility clustering. Therefore, for stationary time series, like financial returns, the extremal index $\theta \in (0,1]$ summarizes short-term extremal behaviour during a stress period, where the level of extremal dependence increases with decreasing $\theta$. The extremal index is related to a measure of clustering of upcrossings, the upcrossings index $\eta$, through $\theta = \frac{\nu}{\tau}\eta$, where $\nu$ is the limiting mean number of upcrossings of high thresh-

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olds and $\tau$ is the limiting mean number of exceedances of the same thresholds (Ferreira, 2006).

We shall assume throughout the paper that we are working with a strictly stationary sequence of random variables, $\mathbf{X} = \{X_n\}_{n\geq 1}$, and that $\mathbf{u} = \{u_n\}_{n\geq 1}$ denotes a sequence of high levels. Notice that if $\mathbf{X}$ satisfies the mixing $\Delta(\mathbf{u})$ condition of Hsing *et al.* (1988) and the local dependence condition $D''(\mathbf{u})$ of Leadbetter and Nandagopalan (1988), that locally restricts the dependence of the sequence but allows clustering of exceedances, we have $\eta = 1$. Nevertheless, condition $D''(\mathbf{u})$ doesn't hold for certain autoregressive processes and consequently they possess an upcrossings index $\eta \in (0,1)$ (Ferreira, 2006, 2007, and Sebastião *et al.*, 2010). The upcrossings index can also be defined as the reciprocal of the limiting mean size of a cluster of upcrossings. Hence, identifying clusters of high level upcrossings is a key issue for the estimation of $\eta$. If a sequence is suitably well-behaved then one might hope that groups of successive upcrossings of high levels are sufficiently far apart that each group can be regarded as a separate cluster. A sufficient condition for this to hold is condition $\widetilde{D}^{(3)}(\mathbf{u})$ of Ferreira (2006) which states that

$$\lim_{n\to\infty} nP(X_1 \leq u_n < X_2,\ \widetilde{N}_{3,3} = 0,\ \widetilde{N}_{4,r_n} > 0) = 0,$$

where $\widetilde{N}_{i,j} = \widetilde{N}_n([i/n, j/n]) = \sum_{i=1}^{n} \mathrm{I\!I}_{\{X_i \leq u_n < X_{i+1}\}} \delta_{\frac{i}{n}}([i/n, j/n])$ and $r_n = [n/k_n]$ with $\{k_n\}_{n\geq 1}$ satisfying $k_n \xrightarrow[n\to+\infty]{} +\infty$, $k_n l_n/n \xrightarrow[n\to+\infty]{} 0$ and $k_n \alpha_{n,l_n} \xrightarrow[n\to+\infty]{} 0$, being $\alpha_{n,l_n}$ the mixing coefficients of the $\Delta(\mathbf{u})$ condition.

This condition locally also restricts the dependence of the sequence, but allows clustering of upcrossings contrarily to condition $D''(\mathbf{u})$. It roughly states that whenever an upcrossing of a high level occurs, a cluster of upcrossings may follow it, but once the sequence doesn't upcross the threshold it is very unlikely to upcross it again in the nearby observations. Thus, it enables the identification of upcrossings clusters by the occurrence of an upcrossing followed by a non-upcrossing. Indeed, Ferreira (2007) proved that if the conditional upcrossings run length distribution is defined, for each $j \geq 1$, as

$$\widetilde{\pi}_n(j) = P\left(\bigcap_{i=1}^{j}\{X_{2i+1} \leq u_n < X_{2i+2}\}, \widetilde{N}_{2j+3,2j+3} = 0 \,\middle|\, \widetilde{N}_{1,1} = 0, X_3 \leq u_n < X_4\right),$$

then we have

$$\begin{aligned}\eta &= \frac{1}{\lim\limits_{n\to+\infty} \sum\limits_{j\geq 1} j\widetilde{\pi}_n(j)} \\ &= \lim_{n\to+\infty} P(\widetilde{N}_{3,3} = 0 \mid X_1 \leq u_n < X_2), \qquad (2.12)\end{aligned}$$

where $\{u_n\}_{n\geq 1}$ is a sequence of values such that $nP(X_1 \leq u_n < X_2) \xrightarrow[n\to+\infty]{} \nu$, $\nu > 0$, a so called normalized level for upcrossings.

Under condition $\Delta(\mathbf{u})$ we have asymptotic independence of upcrossings over disjoint blocks of size $r_n$ as proven in Ferreira (2006). Hence, if $\mathbf{X}$ has upcrossings index $\eta > 0$ then it can be easily shown that for normalized levels $u_n$ for upcrossings

$$\eta = \lim_{n\to+\infty} \frac{P(\widetilde{N}_n([0, r_n/n]) > 0)}{r_n P(X_1 \leq u_n < X_2)}. \tag{2.13}$$

The validity of (2.12) and (2.13) provide obvious estimators of $\eta$ that we formally define in the following section.

# 2. Estimating the upcrossings index

For a given sample $(X_1, \ldots, X_n)$, (2.12) provides the obvious estimator of $\eta$ given by the ratio between the number of upcrossings followed by a non-upcrossing and the number of upcrossings of a high threshold, that is, the estimator

$$\hat{\eta}_n^R = \hat{\eta}_n^R(u) := \frac{\sum_{i=1}^{n-3} \mathrm{1\!I}_{\{X_i \leq u < X_{i+1},\ \widetilde{N}_{i+2,i+2}=0\}}}{\sum_{i=1}^{n-1} \mathrm{1\!I}_{\{X_i \leq u < X_{i+1}\}}},$$

where $u$ is a suitable threshold. We shall call this estimator the runs upcrossings index estimator.

Attending to (2.13) we propose the following blocks estimator for $\eta$

$$\hat{\eta}_n^B = \hat{\eta}_n^B(u) := \frac{\sum_{i=1}^{k} \mathrm{1\!I}_{\{\widetilde{N}_{(i-1)r+1,ir}>0\}}}{\sum_{i=1}^{kr-1} \mathrm{1\!I}_{\{X_i \leq u < X_{i+1}\}}},$$

where $k$ is the number of blocks of size $r$ into which the sample is divided and $u$ is a suitable threshold.

The performance of the above mentioned estimators shall be assessed through the analysis of the daily log-returns of the Portuguese PSI20 index from January 2000 until March 2010, among other financial data sets.

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# HILL BIAS OF STUDENT AND STABLE ALTERNATIVES

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## 1. Introduction

The unconditional distribution of financial asset returns are heavy-tailed. There is a long standing debate as to whether this property is better captured by the infinite variance Stable distributions or the finite variance Student-$t$ distributions. In either case, the shape of the distribution, or how heavy the returns are, is measured by the tail index which can be estimated by the popular estimator of Hill (1975). We know from Feller (1975) that heavy-tailed random variables will be additive in the tail region. This property is known as the Feller convolution theorem and is shared by both models. Stable distributions are advantageous in that they are self scaling throughout their domain, but have the disadvantage of an infinite variance. The latter property squares badly with the finite variance view widely held in financial literature.

Both advocates of the sum Stable and the Student-$t$ report Hill estimators as evidence for their respective models. Selecting data using different estimates of the tail size can support either model. As we explain, this is due to different signs of the bias. The paper proposes to use the bias inherent to the Hill estimator to discriminate between the symmetric infinite variance Stable distribution (with a well defined mean) and the finite variance Student-$t$ distribution, which has contributed to much confusion in the literature. The paper also gives the explanation for the hump shape in the Stable distributions, by extending the tail expansion to higher orders.

Suppose the tails of the distribution are regularly varying at infinity and satisfy the following expansion,

$$F(x) = 1 - C_1 x^{-\alpha_1} \left[ 1 + \sum_{i=2}^{N} C_i x^{-\alpha_i} + o\left(x^{-\alpha_N}\right) \right], \ \alpha_i > 0, \text{ as } x \to \infty. \quad (2.14)$$

Both the symmetric Stable distributions with $\alpha_1 \in (1,2)$ and the Student distributions with degrees of freedom $\alpha_1 > 2$ satisfy this expansion. For $i = 2, ..., N$, and with the following parameters:

| | Stable | Student |
|---|---|---|
| $\alpha_1$ | $\in (1,2)$ | $> 2$ |
| $\alpha_i$ | $(i-1)\alpha_1$ | $2(i-1)$ |
| $C_1$ | $\frac{1}{\pi}\Gamma(\alpha_1)\sin\left(\frac{\alpha_1\pi}{2}\right)$ | $\frac{1}{\sqrt{\alpha_1\pi}} \frac{\Gamma\left(\frac{\alpha_1+1}{2}\right)}{\Gamma\left(\frac{\alpha_1}{2}\right)} \alpha_1^{(\alpha_1-1)/2}$ |
| $C_i$ | $\frac{(-1)^{i-1}}{i} \frac{\Gamma(i\alpha_1)}{\Gamma(\alpha_1)} \frac{\sin\left(\frac{i\alpha_1\pi}{2}\right)}{\sin\left(\frac{\alpha_1\pi}{2}\right)}$ | $\frac{(-1)^{i-1}\alpha_1^i}{(i-1)} \frac{(\alpha_1+1)...(\alpha_1+2i-3)}{2^{i-1}(\alpha_1+2i-2)}$ |
| $C$ sign | '+' as $\alpha_1 \in \left(\frac{2(i-1)}{i}, 2\right)$ | '+' as i is odd, '-' as i is even |

Let $\{X_1, ..., X_n\}$ be a sample of i.i.d. r.v.'s with common c.d.f. $F(x)$ which satisfies equation (2.14). Consider the descending order statistics from this sample around a given threshold $s : X_{(1)} \geq ... \geq X_{(m)} > s \geq X_{(m+1)} \geq ... \geq X_{(n)}$. The Hill estimator and the corresponding asymptotic bias are given as follows

$$\widehat{\frac{1}{\alpha_1}} = \frac{1}{m} \sum_{i=1}^{m} \left( \log \frac{X_i}{s} \right)$$

$$E\left[ \widehat{\frac{1}{\alpha_1}} - \frac{1}{\alpha_1} \right] = - \sum_{i=2}^{N} \frac{C_i \alpha_i}{\alpha_1 (\alpha_1 + \alpha_i)} s_n^{-\alpha_i} + o\left(s_n^{-\alpha_N}\right).$$

where $s_n^{\alpha}/n \to 0$, $s_n \to \infty$ as $n \to \infty$.[21]

It can be shown immediately that the sign of the second order scale parameter $C_2$ determines the sign of the bias when $N = 2$. In particular, the estimate of the tail index $\alpha_1$ is downward biased for the Student-*t* and upward biased for the Stable distributions.

When $N = 3$, note that for the Stable class $C_3 = 0$ when $\alpha_1 = \frac{4}{3}$, and $C_3 > 0$ when $\alpha_1 > \frac{4}{3}$, hence the asymptotic bias is larger than that of only considering

[21] The optimal threshold $s$ is chosen such that both parts in asymptotic mean square error (AMSE) vanish at the same rate, i.e. $\partial AMSE/\partial s_n = 0$.

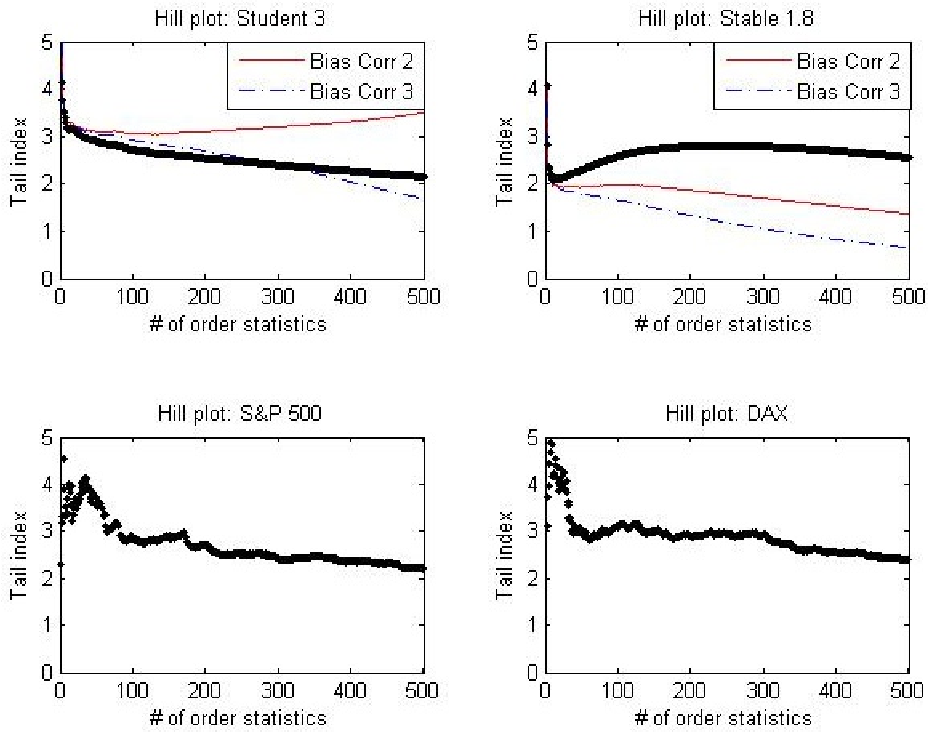


Figure 2.14: Hill & Bias Plot, S&P 500 and DAX (2001-2010)

the second order expansion. This can partially explain the "hump" shape of the Hill plot for $\alpha_1$ close to 2. In the third case of $C_3 < 0$, we have $\alpha_1 < \frac{4}{3}$. The opposite sign of the third scale parameter from the second, in this case, offsets the second order bias term. For the Student model, it is always the case that $C_3 > 0$ and is relatively insignificant in comparison to $C_2$ close to the tail. Therefore, it does not reverse the downward bias that we see in the Hill plot.

This results can be extended to higher orders. For the Stable distribution, as $\alpha_1$ goes to 2, the first $N$ order scale parameters share the positive sign, which enhance the upward bias of Hill estimator, and consequently also the shape of the hump.

# 3. Conclusion

To summarize, under the assumption of either the symmetric Stable or Student-$t$ distribution, the optimal number of order statistics for the Hill estimator is very low. We consistently find that less than 2% of data is required, in contrast what one sees some literature of stable models. When applying this to real financial data, estimates of the tail index are generally significantly larger than 2. This contradicts the assumption of infinite variance of the stable distribution. Unlike the Student-$t$, the stable model has two regions that share the true value of the estimate due to the "hump" shape. Therefore, the "correct" estimate can be found using the Stable distribution even though incorrect tail size is used.

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# ALTERNATIVE MODELING FOR LONG TERM RISK

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**Abstract:** In this paper, we discuss an alternative approach for long term risk modeling and propose four dynamic risk measures. We use long range dependence of some financial data to forecast long-term returns and volatility. Dynamic risks are evaluated by the forecasting and show the picture of risk evolution. The empirical studies implemented with equity data estimate long term market risk. Results indicate that our method gives better evaluations of long term risks than square-root method when there are long term processes. Moreover, non-linear models outperform in the long run, and model with NIG and mixed GPD innovations give more prudent results than others.

## 1. Introduction

Although long term risk measure is not as forthwith required as 1-day risk measure, its importance has been attracting peoples' attention, especially after crisis. It can be reflected in the 1996 Basel Committee. Long term risk measurement, however, remains an unsolved issue. In this paper we intent to find a dynamic way to measure risks within a large time horizon.

The two conventional approaches for long term risk evaluation are square root method and low frequency data method. Unfortunately, the first method has been proved to overestimate risks in long term and is model dependent. The second method suffers from data limitation. People proposed to mix these two approaches in order to overcome the drawback of square root method. But it is still a bias-variance tradeoff problem. In this paper, we suggest an alternative approach for long term risk estimation focusing on long term volatility forecasting and dynamic risk measures.

We expect that long range dependency exist in financial data (Cheung and Lai (1995)) and is a predictable component in dynamic series. We give below the generalized formula for underlying processes, which captures the persistence, intermediate shock and heteroscedasticity property of financial data. If $y_t$ represent the returns, then

$$\begin{aligned}
&\Phi(B)\,\Pi_{i=1}^{k}(I-2\nu_i B+B^2)^{d_i}y_t=\Theta(B)\,\varepsilon_t\\
&\varepsilon_t=h_t\xi_t\\
&h_t^{\delta}=a_0+\sum_{j=1}^{r}a_j\varepsilon_{t-j}^{\delta}+\sum_{f=1}^{s}b_f h_{t-f}^{\delta}+\sum_{l=1}^{v}c_l\varepsilon_{t-l}^{\delta/2}h_{t-l}^{\delta/2},
\end{aligned}\tag{2.15}$$

Here, $\Phi(B)$ and $\Theta(B)$ are ARMA operators, $B$ is the backshift operator, $d=(d_1,\ldots,d_k)$ are memory parameters and $\nu=(\nu_1,\ldots,\nu_k)$ are frequency location parameters. $\xi_t$ is a sequence of *iid rvs* with zero mean and unit variance. $\delta\in\mathbb{R}$, $a_0$, $a_j$ and $b_f$ are real numbers. We test the performance of risk evaluation with five distributions for $\xi_t$: Normal, Student, NIG, GPD, and a mixture of GPD and Elliptical distributions. In empirical study, we experiment five special cases of formula (2.15), and compare performance of linear models, non-linear models, short and long memory processes. We use Whittle estimation method (Robinson P. and Henry M. (1999)) to estimate long memory parameters. We apply iterative maximum likelihood method based on maximization by parts method (Fan Y., Pastorello S. and Renault E. (2007)) for the volatility parameters' estimation.

Then, we propose dynamic approaches for four popular risk measures: Value at Risk ($VaR_t$), Expected Shortfall ($ES_t$), MAximum Loss ($MAL_t$) and MAximum Drawdown ($MAD_t$). Firstly, we compute 1-day advanced value based on the forecasting of returns and volatility:

$$\begin{aligned}
&VaR_{t,q}=-\mu_t-h_t z_q,\\
&ES_{t,q}=\mu_t+h_t E(\varepsilon_t|\varepsilon_t<z_q)\\
&MaL_T=\min\{0,\min_{t=1,\ldots,T}\{\sum_{i=1}^{t}y_i\}\},\\
&MaD_T=\min_{i=1,\ldots,t;t=1,\ldots,T}\{\sum_{j=1}^{t}y_j\}
\end{aligned}$$

Here, $\mu_t$ is the level of the return processes, $h_t$ is defined in formula (2.15), $z_q$ is the $q-$th quantile of $\xi_t$. For long term risk measures:

1. We can use forecasting for long term *ES*, *MAL* and *MAD* estimation.

2. For the long term *VaR*, we face the problem of subadditivity of this incoherent risk measure. Nevertheless, we also provide some results by using Monte Carlo simulation.

Here, we extend the work of Guégan and Tarrant (2010) in a set of dynamic risk measures. The empirical studies are implemented with equity index and individual stock prices from 1990-2010. In summary, we find out that the long memory property is more perceptible in volatility than in returns. Second, comparison of the evolution pictures of 1-day advanced dynamic risks indicates that non-linear model with long memory processes has a better evaluation of 1-day ahead risk in the distant time. Additionally, the NIG and mixed GPD innovations give more prudent estimations of risk than other distributions. Moreover, when we compare the long term risk measures for 1-year, our empirical results affirm our expectation that the long memory process help the prediction of long term risks and give a better evaluation of long term risk than the square root method. We can notice that our model is rely on the existence of long memory property of financial data. An the end, we conclude our study and discuss the advantages and disadvantages of our approach.

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