# Asymptotic comparison at optimal levels of reduced-bias extreme value index estimators*

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**Abstract**

In this paper we are interested in the asymptotic comparison, at optimal levels, of a set of semi-parametric reduced-bias extreme value index estimators, valid for a wide class of heavy-tailed models, underlying the available data. Again, as in the classical case, there is not any estimator that can always dominate the alternatives, but interesting clear-cut patterns are found. Consequently, and in practice, a suitable choice of a set of extreme value index estimators will jointly enable us to better estimate the extreme value index $\gamma$, the primary parameter of extreme or even rare events.

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## 1 The estimators under study and scope of the paper

Let us consider a sample of size $n$ of independent, identically distributed (i.i.d.) random variables (r.v.'s), $X_1, X_2, \ldots, X_n$, with a common distribution function (d.f.) $F$. Let us denote by $X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n}$ the associated ascending order statistics (o.s.) and let us assume that there exist sequences of real constants $\{a_n > 0\}$ and $\{b_n \in \mathbb{R}\}$ such that the maximum, linearly

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normalized, i.e., $(X_{n:n} - b_n)/a_n$, converges in distribution towards a non-degenerate r.v. Then the limit distribution is necessarily of the type of the general *extreme value* (EV) d.f., given by

$$EV_\gamma(x) = \begin{cases} \exp(-(1+\gamma x)^{-1/\gamma}), \ 1+\gamma x > 0 & \text{if} \quad \gamma \neq 0 \\ \exp(-\exp(-x)), \ x \in \mathbb{R} & \text{if} \quad \gamma = 0. \end{cases} \tag{1.1}$$

The d.f. $F$ is said to belong to the max-domain of attraction of $EV_\gamma$, and we write $F \in \mathcal{D}_{\mathcal{M}}(EV_\gamma)$. The parameter $\gamma$ is the *extreme value index* (EVI), the primary parameter of extreme events. This index measures the heaviness of the right *tail function* $\overline{F} := 1 - F$, and the heavier the right tail, the larger $\gamma$ is. In this paper we shall work with heavy-tailed models, i.e., Pareto-type underlying d.f.'s, with a strict positive EVI. These heavy-tailed models are quite common in the most diversified areas of application, like computer science, telecommunications, insurance, finance, bibliometrics and biostatistics, among others. Also very popular in insurance and finance are all right tails of the type, $\overline{F}(x) = 1 - F(x) = \exp\{-H(x)\}$, $H \in RV_{1/\theta}$, $\theta > 1$, with $RV_\alpha$ denoting the class of regularly varying functions at infinity, with an index of regular variation equal to $\alpha$ (see Bingham *et al.*, 1987). In a context of *extreme value theory* we have then a null EVI, i.e., $\gamma = 0$, in (1.1), but we are working with those tails in the domain of attraction for maxima of $EV_0(\cdot)$, which exhibit a penultimate behaviour (see Fisher and Tippett, 1928, Gomes, 1984, and Diebolt and Guillou, 2005, among others), looking more similar to Pareto tails than to exponential tails. These distributions belong to the class of sub-exponential models, another possible class of heavy-tailed models.

For heavy-tailed models in $\mathcal{D}_{\mathcal{M}}^{+} \equiv \mathcal{D}_{\mathcal{M}}(EV_\gamma)_{\gamma>0}$, the classical EVI-estimators are the Hill estimators (Hill, 1975), which are the average of the scaled log-spacings as well as of the log-excesses, given by

$$U_i := i\left\{\ln \frac{X_{n-i+1:n}}{X_{n-i:n}}\right\} \qquad \text{and} \qquad V_{ik} := \ln \frac{X_{n-i+1:n}}{X_{n-k:n}}, \quad 1 \le i \le k < n, \tag{1.2}$$

respectively. We have

$$H_n(k) \equiv H(k) = \frac{1}{k}\sum_{i=1}^{k} U_i = \frac{1}{k}\sum_{i=1}^{k} V_{ik}, \quad 1 \le k < n. \tag{1.3}$$

But the EVI-estimators in (1.3) have often a strong asymptotic bias for moderate up to large values of $k$. Consequently, the adequate accommodation of the bias of Hill's estimators has been extensively addressed in recent years by several authors. We mention the pioneering papers by Peng (1998), Beirlant *et al.* (1999), Feuerverger and Hall (1999) and Gomes *et al.* (2000). In all these papers, authors are led to second-order reduced-bias EVI-estimators, with asymptotic variances always larger or equal to $(\gamma\ (1-\rho)/\rho)^2$, the minimal asymptotic variance of an "asymptotically unbiased" estimator in Drees' class of functionals (Drees, 1998), where $\rho < 0$ is a shape second-order parameter ruling the rate of convergence of the normalized sequence of maximum values towards the limiting law $EV_\gamma$, in (1.1). Recently, Caeiro *et al.* (2005) and Gomes *et al.* (2007*a*; 2008) considered, in different ways, the problem of corrected-bias EVI-estimation, being able to reduce the bias without increasing the asymptotic variance, which was shown to be kept at the value $\gamma^2$, the asymptotic variance of Hill's estimator, the maximum likelihood estimator of $\gamma$ for an underlying Pareto d.f., $F_{_P}(x) = 1 - (x/C)^{-1/\gamma},\ x \geq C$. Those estimators, called *minimum-variance reduced-bias* (MVRB) EVI-estimators, are all based on an adequate external estimation of a pair of second-order parameters, $(\beta, \rho) \in (\mathbb{R}, \mathbb{R}^-)$, done through estimators denoted now $(\hat\beta, \hat\rho)$. Gomes *et al.* (2008) considered the first estimator of this type, an EVI-estimator based on a linear combination of the log-excesses $V_{ik}$, in (1.2), given by

$$\overline{WH}_{\hat\beta,\hat\rho}(k) := \frac{1}{k}\sum_{i=1}^{k} e^{-\hat\beta\ (n/k)^{\hat\rho}\ \psi_{ik}(\hat\rho)}\ V_{ik}, \qquad \psi_{ik}(\rho) = \psi_{ik} = -\frac{(i/k)^{-\rho}-1}{\rho\ln(i/k)}, \tag{1.4}$$

*WH* standing here for *weighted Hill* estimator. Caeiro *et al.* (2005) considered two estimators of this same type, here denoted,

$$CH_{\hat\beta,\hat\rho}(k) := H(k)\Big(1 - \frac{\hat\beta}{1-\hat\rho}\Big(\frac{n}{k}\Big)^{\hat\rho}\Big) \quad \text{and} \quad \overline{CH}_{\hat\beta,\hat\rho}(k) := H(k)\exp\Big(-\frac{\hat\beta}{1-\hat\rho}\Big(\frac{n}{k}\Big)^{\hat\rho}\Big). \tag{1.5}$$

A third class was introduced in Gomes *et al.* (2007a), and it has the functional form

$$ML_{\widehat\beta,\widehat\rho}(k) := H(k) - \widehat\beta\ \Big(\frac{n}{k}\Big)^{\widehat\rho}\ \Big(\frac{1}{k}\sum_{i=1}^{k}\Big(\frac{i}{k}\Big)^{-\widehat\rho} U_i\Big), \tag{1.6}$$

with $U_i$ given in (1.2). These authors considered also the estimator

$$\overline{ML}_{\widehat\beta,\widehat\rho}(k) := \frac{1}{k}\sum_{i=1}^{k}\exp\big(-\hat\beta(n/i)^{\hat\rho}\big)U_i, \tag{1.7}$$

the estimator directly derived from the likelihood equation for $\gamma$ with $\beta$ and $\rho$ fixed and based upon the exponential approximation $U_i \approx \gamma\exp(\beta(n/i)^\rho)E_i$, $1 \leq i \leq k$, with $\{E_i\}_{i\geq 1}$ denoting a sequence of independent, standard exponential r.v.'s, being claimed a better performance of the

$ML$ estimator, comparatively to the $\overline{ML}$ estimator, for a large class of models. This is the reason why we shall also work with a first order approximation for the estimator $\overline{WH}_{\hat\beta,\hat\rho}(k)$, in (1.4), the bias-corrected Hill estimator

$$WH_{\hat\beta,\hat\rho}(k) := H(k) - \hat\beta\ \Big(\frac{n}{k}\Big)^{\hat\rho}\Big(\frac{1}{k}\sum_{i=1}^{k}\psi_{ik}\ V_{ik}\Big), \tag{1.8}$$

with $\psi_{ik}$ given in (1.4). For the estimation of $\rho$ it has been advised the use of the simplest class of estimators in Fraga Alves *et al.* (2003), denoted now $\hat\rho(k)$, which are computed at $k_1 = n^{1-\epsilon}$, for any small value $\epsilon$, like $\epsilon = 0.01$. In the notation of this paper, $\hat\rho = \hat\rho(k_1)$. The estimation of $\beta$ has been performed through the use of a statistic dependent on a consistent estimator of $\rho$, denoted $\hat\beta(k;\hat\rho)$, introduced in Gomes and Martins (2002), and also computed at the same $k_1$, i.e., $\hat\beta = \hat\beta(k_1;\hat\rho)$.

Apart from any of the above mentioned MVRB statistics, denoted generally $UH_{\hat\beta,\hat\rho}(k)$, with $UH$ standing for *unbiased Hill*, we shall also consider the statistics

$$UH^*_{\hat\rho}(k) := UH_{\hat\beta(k;\hat\rho),\hat\rho}(k), \tag{1.9}$$

with an asymptotic variance that is no longer $\gamma^2$ but $\gamma^2(1-\rho)^2/\rho^2 > \gamma^2$ for every $\rho$. As mentioned before, in the $UH_{\hat\beta,\hat\rho}(k)$ and $UH^*_{\hat\rho}(k)$ classes, $(\hat\beta,\hat\rho)$ or $\hat\rho$, respectively, need to be adequate consistent estimators of the second-order parameters, if we want to keep the same properties of $UH_{\beta,\rho}(k)$ or $UH^*_{\rho}(k)$, the associated r.v.'s. For details on such a type of estimation see, for instance, Caeiro and Gomes (2008), Caeiro *et al.* (2009) and and Gomes *et al.* (2009), among others.

In this paper, we compare asymptotically, at optimal levels, the above mentioned MVRB statistics, denoted generically $UH(k)$, and the reduced-bias statistics $UH^*(k)$ (assuming thus that $\beta$ and $\rho$ are known or adequately estimated). In Section 2, we provide a brief review of the most common first, second and third-order frameworks for heavy-tailed models. In Section 3, we derive, for a large class of models in $\mathcal{D}^+_{\mathcal{M}}$, the asymptotic properties of $UH(k)$ and $UH^*(k)$, and finally, in Section 4, we provide a full asymptotic comparison, at optimal levels, of $UH(k)$ and $UH^*(k)$ for $UH = CH, ML$ and $WH$.

## 2 A brief review of first, second and third-order conditions for heavy tails

In the most common frameworks in the area of *statistics of extremes*, a model $F$ is said to be *heavy-tailed* whenever the tail function $\overline{F}$ is a *regularly varying function* with a negative index of regular variation equal to $\{-1/\gamma\}$, $\gamma > 0$, or equivalently, the reciprocal quantile function $U(t) = F^{\leftarrow}(1-1/t),\ t \geq 1$, with $F^{\leftarrow}(x) = \inf\{y : F(y) \geq x\}$, is of regular variation with index $\gamma$, i.e., for all $x > 0$,

$$\lim_{t\to\infty} \frac{\overline{F}(tx)}{\overline{F}(t)} = x^{-1/\gamma} \quad \Longleftrightarrow \quad \lim_{t\to\infty} \frac{U(tx)}{U(t)} = x^{\gamma} \quad \Longleftrightarrow \quad F \in \mathcal{D}_{\mathcal{M}}^{+}. \tag{2.1}$$

The *second-order parameter*, $\rho$ $(\leq 0)$, rules the rate of convergence in the first-order condition (2.1), and is the non-positive parameter appearing in the limiting relation

$$\lim_{t\to\infty} \frac{\ln U(tx) - \ln U(t) - \gamma \ln x}{A(t)} = \frac{x^{\rho}-1}{\rho}, \tag{2.2}$$

which is assumed to hold for every $x > 0$, and where $|A|$ must then be of regular variation with index $\rho$ (Geluk and de Haan, 1987).

In order to obtain information on the order of the asymptotic bias of second-order reduced-bias EVI-estimators, it is necessary to further assume a third-order condition, ruling the rate of convergence in (2.2), and which guarantees that, for all $x > 0$,

$$\lim_{t\to\infty} \frac{\frac{\ln U(tx) - \ln U(t) - \gamma \ln x}{A(t)} - \frac{x^{\rho}-1}{\rho}}{B(t)} = \frac{x^{\rho+\rho'}-1}{\rho+\rho'}, \tag{2.3}$$

where $|B(t)|$ must then be of regular variation with index $\rho'$. There appears then this extra non-positive third-order parameter $\rho' \leq 0$. Such a condition has already been used in Gomes *et al.* (2002) and Fraga Alves *et al.* (2003), for the full derivation of the asymptotic behaviour of $\rho$-estimators, in Gomes *et al.* (2004), for the study of a reduced-bias EVI-estimator and more recently in Caeiro *et al.* (2009), for a comparison of some of the MVRB estimators in this paper.

In this paper, similarly to what has been done in Gomes *et al.* (2007a), we consider a Pareto-type class of models, with a tail function

$$1 - F(x) = Cx^{-1/\gamma}\big(1 + D_1 x^{\rho/\gamma} + D_2 x^{2\rho/\gamma} + o\big(x^{2\rho/\gamma}\big)\big), \tag{2.4}$$

as $x \to \infty$, with $C > 0$, $D_1,\ D_2 \neq 0$, $\rho < 0$. Note that to assume (2.4) is equivalent to say that (2.3) holds with $\rho = \rho' < 0$ and that we may there choose

$$A(t) = \alpha\ t^\rho =: \gamma\ \beta\ t^\rho, \qquad B(t) = \beta'\ t^\rho = \frac{\beta' A(t)}{\beta\gamma}, \qquad \beta,\ \beta' \neq 0, \tag{2.5}$$

with $\beta$ and $\beta'$ "scale" second and third-order parameters, respectively.

**Remark 2.1.** *Several common heavy-tailed models belong to the class in* (2.4). *Among them we mention:*

- *the* Fréchet *model, with d.f.* $F(x) = \exp(-x^{-1/\gamma})$, $x \geq 0$, $\gamma > 0$, *for which* $\rho' = \rho = -1$, $\beta = 0.5$ *and* $\beta' = 5/6$*;*
- *the* Generalized Pareto (*GP*) *model, with d.f.* $F(x) = 1 - (1 + \gamma x)^{-1/\gamma}$, $x \geq 0$, $\gamma > 0$, *for which* $\rho' = \rho = -\gamma$ *and* $\beta = \beta' = 1$*;*
- *the* Burr *model, with d.f.* $F(x) = 1 - (1 + x^{-\rho/\gamma})^{1/\rho}$, $x \geq 0$, $\gamma > 0$, $\rho' = \rho < 0$ *and, as for the GP model,* $\beta = \beta' = 1$*;*
- *the* Student*'s* $t_\nu$*-model with* $\nu$ *degrees of freedom, with a probability density function* (*p.d.f.*)

  $$f_{t_\nu}(t) = \Gamma((\nu+1)/2)\left[1 + t^2/\nu\right]^{-(\nu+1)/2} / (\sqrt{\pi\nu}\ \Gamma(\nu/2)),\ t \in \mathbb{R} \quad (\nu > 0),$$

  *for which* $\gamma = 1/\nu$ *and* $\rho' = \rho = -2/\nu$. *Regarding the values of* $(\beta, \beta')$, *we provide* Table 1. *For an explicit expression of* $\beta$ *and* $\beta'$ *as a function of* $\nu$, *see* Caeiro and Gomes (2008).

Table 1: Values of $\beta$ and $\beta'$ for a Student d.f with $\nu$ degrees of freedom.

| $\nu$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $\beta$ | 6.580 | 3.000 | 2.249 | 1.925 | 1.742 | 1.625 | 1.542 | 1.480 |
| $\beta'$ | 4.606 | 2.334 | 1.848 | 1.636 | 1.516 | 1.439 | 1.385 | 1.344 |
| $\xi := \beta'/\beta$ | 0.700 | 0.778 | 0.821 | 0.850 | 0.870 | 0.886 | 0.898 | 0.907 |

In order to have consistency of any of the EVI-estimators mentioned in Section 1, in all $\mathcal{D}_{\mathcal{M}}^{+}$, we need to work with intermediate values of $k$, i.e., a sequence of integers $k = k_n$, $1 \leq k < n$, such that

$$k = k_n \to \infty \quad \text{and} \quad k_n = o(n), \ \text{as}\ n \to \infty. \tag{2.6}$$

Under the second order framework, in (2.2), the asymptotic distributional representation

$$H_n(k) \ \stackrel{d}{=} \ \gamma + \frac{\gamma}{\sqrt{k}}\, Z_k + \frac{1}{1-\rho}\ A(n/k)(1+o_p(1))$$

holds (de Haan and Peng, 1998), where, with $\{E_i\}$ a sequence of i.i.d. standard exponential r.v.'s,

$$Z_k = \sqrt{k}\Big(\sum_{i=1}^{k} E_i/k - 1\Big) \tag{2.7}$$

is an asymptotically standard normal r.v. Under adequate conditions on $k$, we get, for any of the above mentioned MVRB statistics, in (1.4), (1.5), (1.6), (1.7) and (1.8), denoted generally $UH_{\hat\beta,\hat\rho}(k)$, the validity of the asymptotic distributional representation

$$UH_{\hat\beta,\hat\rho}(k) \ \stackrel{d}{=} \ \gamma + \frac{\gamma}{\sqrt{k}}\, Z_k + o_p\big(A(n/k)\big).$$

Under similar conditions, and for any of the reduced-bias statistics in (1.9), we get

$$UH^*_{\hat\rho}(k) \ \stackrel{d}{=} \ \gamma + \frac{\gamma(1-\rho)}{\rho\sqrt{k}}\ Z_k + o^*_p\big(A(n/k)\big).$$

Detailed information on the remainder terms $o_p(A(n/k)$ and $o^*_p(A(n/k))$ will be given next, in Section 3.

## 3 The asymptotic behaviour of $UH$ and $UH^*$

We now state the following result, a particular case, with some additions related with the $\overline{UH}$ statistics, of Theorem 3.1 and Theorem 3.2 in Caeiro *et al.* (2009).

**Theorem 3.1.** *Under the third-order framework in* (2.4)*, with* $A(t)$ *given in* (2.5)*,* $Z_k$ *given in* (2.7)*, and for intermediate* $k$*, i.e., if* (2.6) *holds, we can write*

$$UH_{\beta,\rho}(k) \ \stackrel{d}{=} \ \gamma + \frac{\gamma\ Z_k}{\sqrt{k}} + \Big(b_{_{UH}} A^2(n/k) + O_p\Big(\frac{A(n/k)}{\sqrt{k}}\Big)\Big)(1+o_p(1)),$$

*and*

$$UH^*_{\rho}(k) \ \stackrel{d}{=} \ \gamma + \frac{\gamma(1-\rho)\ Z_k}{|\rho|\sqrt{k}} + \Big(b^*_{_{UH}} A^2(n/k) + O_p\Big(\frac{A(n/k)}{\sqrt{k}}\Big)\Big)(1+o_p(1)),$$

*where, with* $\xi = \beta'/\beta$,

$$b_{_{CH}} = \frac{1}{\gamma}\Big(\frac{\xi}{1-2\rho} - \frac{1}{(1-\rho)^2}\Big), \quad b_{_{\overline{CH}}} = \frac{1}{\gamma}\Big(\frac{\xi}{1-2\rho} - \frac{1}{2(1-\rho)^2}\Big),$$

$$b_{ML} = \frac{\xi - 1}{\gamma(1-2\rho)}, \quad b_{\overline{ML}} = \frac{2\xi - 1}{2\gamma(1-2\rho)},$$

$$b^*_{CH} = -\frac{1}{\gamma(1-3\rho)}\Big(\frac{\xi(1-\rho)}{1-2\rho} - \frac{2\rho^2-\rho+1}{(1-\rho)^2}\Big), \quad b^*_{\overline{CH}} = -\frac{1}{\gamma(1-3\rho)}\Big(\frac{\xi(1-\rho)}{1-2\rho} - \frac{4\rho^2-5\rho+3}{2(1-\rho)^2}\Big),$$

$$b^*_{ML} = -\frac{(1-\rho)(\xi-1)}{\gamma(1-2\rho)(1-3\rho)}, \quad b^*_{\overline{ML}} = -\frac{2\xi(1-\rho)-(3-5\rho)}{2\gamma(1-2\rho)(1-3\rho)},$$

*and with*

$$a_2(\rho) = -\left(\ln(1-2\rho) - 2\ln(1-\rho)\right)/\rho^2,$$

$$b_{WH} = \frac{1}{\gamma}\Big(\frac{\xi}{1-2\rho} - a_2(\rho)\Big), \quad b_{\overline{WH}} = \frac{1}{\gamma}\Big(\frac{\xi}{1-2\rho} - \frac{a_2(\rho)}{2}\Big),$$

$$b^*_{WH} = -\frac{1}{\gamma(1-3\rho)}\Big(\frac{\xi(1-\rho)}{1-2\rho} - 2 + (1-3\rho)a_2(\rho)\Big),$$

$$b^*_{\overline{WH}} = - - \frac{1}{\gamma(1-3\rho)}\Big(\frac{\xi(1-\rho)}{1-2\rho} - \frac{4-(1-3\rho)a_2(\rho)}{2}\Big).$$

*Consequently, even if* $\sqrt{k}\ A(n/k) \to \infty$*, with* $\sqrt{k}\ A^2(n/k) \to \lambda_A$*, finite,*

$$\sqrt{k}\ \ (UH_{\beta,\rho}(k) - \gamma) \ \underset{n\to\infty}{\overset{d}{\longrightarrow}} \ Normal\big(\lambda_A b_{UH}, \sigma^2_{UH} = \gamma^2\big) \tag{3.1}$$

*and*

$$\sqrt{k}\ \ \big(UH^*_\rho(k) - \gamma\big) \ \underset{n\to\infty}{\overset{d}{\longrightarrow}} \ Normal\big(\lambda_A b^*_{UH}, \sigma^2_{UH^*} = \frac{\gamma^2(1-\rho)^2}{\rho^2}\big). \tag{3.2}$$

*If* $\sqrt{k}\ A^2(n/k) \to \infty$*, both* $(UH_{\beta,\rho}(k) - \gamma)$ *and* $\big(UH^*_\rho(k) - \gamma\big)$ *are* $O_p(A^2(n/k))$*.*

*Moreover, if we consistently estimate the vector* $(\beta, \rho)$ *of second-order parameters through* $(\hat\beta, \hat\rho)$*, with* $\hat\rho - \rho = o_p(ln(n/k))/(\sqrt{k}A(n/k))$*, (3.1) and (3.2) still hold, with* $UH_{\beta,\rho}(k)$ *and* $UH^*_\rho(k)$ *replaced by* $UH_{\hat\beta,\hat\rho}(k)$ *and* $UH^*_{\hat\rho}(k)$*, respectively.*

*Proof.* The results for $\overline{UH}$ follow straightforwardly from the results for $UH$, derived in Caeiro *et al.* (2009). Indeed, as $n \to \infty$, we have

$$\overline{WH}_{\beta,\rho} - WH_{\beta,\rho} \overset{p}{\sim} a_2(\rho)A^2(n/k)/(2\gamma),$$

$$\overline{CH}_{\beta,\rho} - CH_{\beta,\rho} \overset{p}{\sim} A^2(n/k)/(2\gamma(1-\rho)^2)$$

and

$$\overline{ML}_{\beta,\rho} - ML_{\beta,\rho} \overset{p}{\sim} A^2(n/k)/(2\gamma(1-2\rho)).$$

■

**Remark 3.1.** *For the adequate estimation of* $(\beta, \rho)$ *see* Gomes and Pestana (2007) *and* Caeiro *et al.* (2009)*, among others.*

**Remark 3.2.** *Note that* $b_{_{ML}} = b_{_{ML^*}} = 0$ *whenever* $\xi = 1$ $(\beta' = \beta)$*. This happens for important models like the Burr and the GP, and it is a point in favour of the* $ML$*-statistic, as already mentioned in* Gomes *et al.* (2007a)*.*

**Remark 3.3.** *Note also that, as already mentioned in* Caeiro *et al.* (2009)*, since* $\lambda_{_A} \geq 0$ *and* $1/(2a_2(\rho)) > (1-\rho)^2 > 1/a_2(\rho) > 1-2\rho$ *for any* $\rho < 0$, $b_{_{\overline{WH}}} \geq b_{_{CH}} \geq b_{_{WH}} \geq b_{_{ML}}$*. All depends then on the sign of the bias, but we expect the sample paths of* $\overline{WH}$ *to be always above the sample paths of* $CH$*, which should be in turn above the sample paths of* $WH$*, these ones above the ones of* $ML$*. The* $ML$*-statistics are then preferable to the other ones whenever the bias are all positive. If the bias are all negative,* $\overline{WH}$ *is expected to outperform the other statistics.*

# 4 Asymptotic comparison of the estimators at optimal levels

Following the comparison done in Gomes *et al.* (2007a) for the MVRB estimators $ML$ and $ML^*$, we shall next proceed to the comparison of the estimators under study at their optimal levels. This is again done in a way similar to the one used in de Haan and Peng (1998), Gomes and Martins (2001), Gomes *et al.* (2005, 2007b), Gomes and Neves (2007) and Gomes and Henriques-Rodrigues (2009) for the classical EVI-estimators. Let us assume that $\widehat{\gamma}_n^\bullet(k)$ denotes any arbitrary reduced-bias semi-parametric estimator of the extreme value index $\gamma$, for which we have

$$\widehat{\gamma}_n^\bullet(k) = \gamma + \frac{\sigma_\bullet}{\sqrt{k}} Z_k^\bullet + b_\bullet\ A^2(n/k) + o_p(A(n/k)) \tag{4.1}$$

for any intermediate sequence of integers $k = k_n$, and where $Z_k^\bullet$ is an asymptotically standard normal r.v. Then, $\sqrt{k}\big(\widehat{\gamma}_n^\bullet(k) - \gamma\big) \xrightarrow{d} N(\lambda_{_A} b_\bullet, \sigma_\bullet^2)$ provided that $k$ is such that $\sqrt{k}\ A^2(n/k) \to \lambda_{_A}$, finite, as $n \to \infty$. We then write $Bias_\infty\big(\widehat{\gamma}_n^\bullet(k)\big) := b_\bullet\ A^2(n/k)$ and $Var_\infty\big(\widehat{\gamma}_n^\bullet(k)\big) := \sigma_\bullet^2/k$. The so-called Asymptotic Mean Square Error ($AMSE$) is then given by

$$AMSE\left(\widehat{\gamma}_n^\bullet(k)\right) := \frac{\sigma_\bullet^2}{k} + b_\bullet^2\ A^4(n/k).$$

Regular variation theory (Bingham *et al.*, 1997), enables us to show that, whenever $b_\bullet \neq 0$, there exists a function $\varphi(n) = \varphi(n, \gamma, \rho)$, such that

$$\lim_{n\to\infty} \varphi(n)\ AMSE\left(\widehat{\gamma}_{n0}^\bullet\right) = \left(\sigma_\bullet^2\right)^{-\frac{4\rho}{1-4\rho}} \left(b_\bullet^2\right)^{\frac{1}{1-4\rho}} =: LMSE\left(\widehat{\gamma}_{n0}^\bullet\right),$$

where $\widehat{\gamma}^{\bullet}_{n0} := \widehat{\gamma}^{\bullet}_{n,k^{\bullet}_0(n)}$ and $k^{\bullet}_0(n) := \arg\inf_{k} AMSE\,(\widehat{\gamma}^{\bullet}_n(k))$.

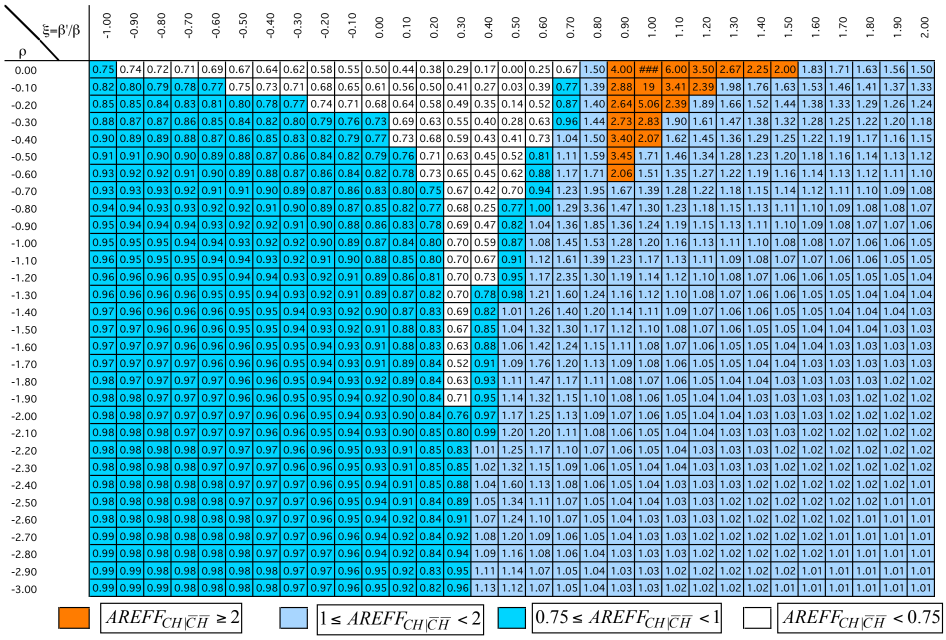
It is then sensible to consider the following:

**Definition 4.1.** *Given two biased estimators $\widehat{\gamma}^{(1)}_n(k)$ and $\widehat{\gamma}^{(2)}_n(k)$, for which a distributional representation of the type of the one in* (4.1) *holds, with constants $(\sigma_1, b_1)$ and $(\sigma_2, b_2)$, $b_1, b_2 \neq 0$, respectively, both computed at their optimal levels, the Asymptotic Root Efficiency ($AREFF$) of $\widehat{\gamma}^{(1)}_{n0}$ relatively to $\widehat{\gamma}^{(2)}_{n0}$ is*

$$AREFF_{1|2} \equiv AREFF_{\widehat{\gamma}^{(1)}_{n0}|\widehat{\gamma}^{(2)}_{n0}} := \sqrt{LMSE\big[\widehat{\gamma}^{(2)}_{n0}\big]/LMSE\big[\widehat{\gamma}^{(1)}_{n0}\big]} = \left(\left(\frac{\sigma_2}{\sigma_1}\right)^{-4\rho}\left|\frac{b_2}{b_1}\right|\right)^{\frac{1}{1-4\rho}}. \tag{4.2}$$

**Remark 4.1.** *Note that the $AREFF$ indicator, in* (4.2)*, has been conceived so that the highest the $AREFF$ indicator is, the better is the first estimator.*

We first present in Figures 1, 2 and 3, the measure $AREFF_{UH|\overline{UH}}$ for $UH = CH, ML, WH$, respectively.

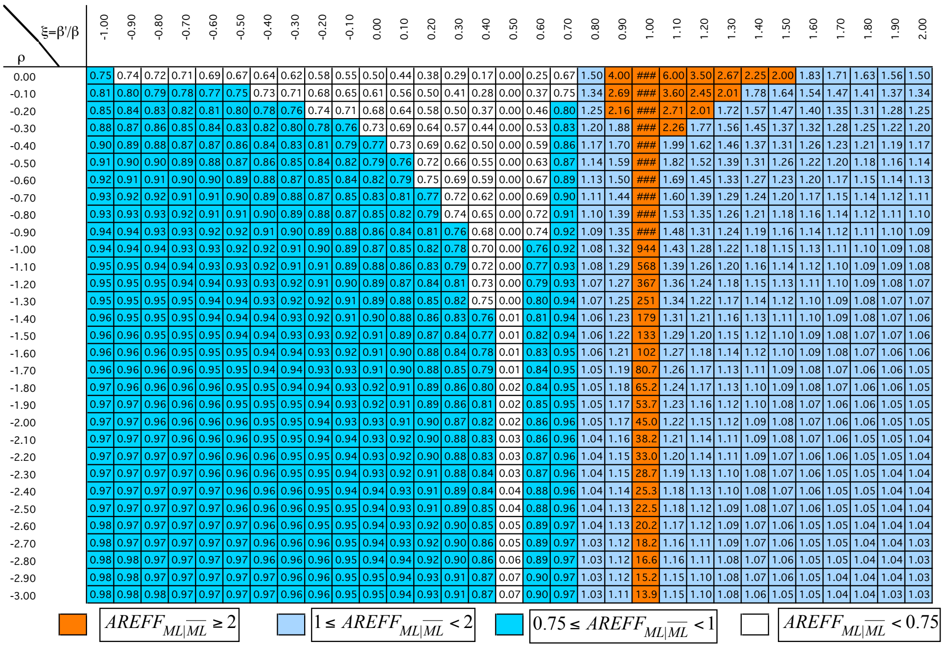


| ξ=β'/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.75 | 0.74 | 0.72 | 0.71 | 0.69 | 0.67 | 0.64 | 0.62 | 0.58 | 0.55 | 0.50 | 0.44 | 0.38 | 0.29 | 0.17 | 0.00 | 0.25 | 0.67 | 1.50 | 4.00 | ### | 6.00 | 3.50 | 2.67 | 2.25 | 2.00 | 1.83 | 1.71 | 1.63 | 1.56 | 1.50 |
| -0.10 | 0.82 | 0.80 | 0.79 | 0.78 | 0.77 | 0.75 | 0.73 | 0.71 | 0.68 | 0.65 | 0.61 | 0.56 | 0.50 | 0.41 | 0.27 | 0.03 | 0.39 | 0.77 | 1.39 | 2.88 | 19 | 3.41 | 2.39 | 1.98 | 1.76 | 1.63 | 1.53 | 1.46 | 1.41 | 1.37 | 1.33 |
| -0.20 | 0.85 | 0.85 | 0.84 | 0.83 | 0.81 | 0.80 | 0.78 | 0.77 | 0.74 | 0.71 | 0.68 | 0.64 | 0.58 | 0.49 | 0.35 | 0.14 | 0.52 | 0.87 | 1.40 | 2.64 | 5.06 | 2.39 | 1.89 | 1.66 | 1.52 | 1.44 | 1.38 | 1.33 | 1.29 | 1.26 | 1.24 |
| -0.30 | 0.88 | 0.87 | 0.87 | 0.86 | 0.85 | 0.84 | 0.82 | 0.80 | 0.79 | 0.76 | 0.73 | 0.69 | 0.63 | 0.55 | 0.40 | 0.28 | 0.63 | 0.96 | 1.44 | 2.73 | 2.83 | 1.90 | 1.61 | 1.47 | 1.38 | 1.32 | 1.28 | 1.25 | 1.22 | 1.20 | 1.18 |
| -0.40 | 0.90 | 0.89 | 0.89 | 0.88 | 0.87 | 0.86 | 0.85 | 0.83 | 0.82 | 0.79 | 0.77 | 0.73 | 0.68 | 0.59 | 0.43 | 0.41 | 0.73 | 1.04 | 1.50 | 3.40 | 2.07 | 1.62 | 1.45 | 1.36 | 1.29 | 1.25 | 1.22 | 1.19 | 1.17 | 1.16 | 1.15 |
| -0.50 | 0.91 | 0.91 | 0.90 | 0.90 | 0.89 | 0.88 | 0.87 | 0.86 | 0.84 | 0.82 | 0.79 | 0.76 | 0.71 | 0.63 | 0.45 | 0.52 | 0.81 | 1.11 | 1.59 | 3.45 | 1.71 | 1.46 | 1.34 | 1.28 | 1.23 | 1.20 | 1.18 | 1.16 | 1.14 | 1.13 | 1.12 |
| -0.60 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.86 | 0.84 | 0.82 | 0.78 | 0.73 | 0.65 | 0.45 | 0.62 | 0.88 | 1.17 | 1.71 | 2.06 | 1.51 | 1.35 | 1.27 | 1.22 | 1.19 | 1.16 | 1.14 | 1.13 | 1.12 | 1.11 | 1.10 |
| -0.70 | 0.93 | 0.93 | 0.93 | 0.92 | 0.91 | 0.91 | 0.90 | 0.89 | 0.87 | 0.86 | 0.83 | 0.80 | 0.75 | 0.67 | 0.42 | 0.70 | 0.94 | 1.23 | 1.95 | 1.67 | 1.39 | 1.28 | 1.22 | 1.18 | 1.15 | 1.14 | 1.12 | 1.11 | 1.10 | 1.09 | 1.08 |
| -0.80 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.89 | 0.87 | 0.85 | 0.82 | 0.77 | 0.68 | 0.25 | 0.77 | 1.00 | 1.29 | 3.36 | 1.47 | 1.30 | 1.23 | 1.18 | 1.15 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 | 1.08 | 1.07 |
| -0.90 | 0.95 | 0.94 | 0.94 | 0.94 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.88 | 0.86 | 0.83 | 0.78 | 0.69 | 0.47 | 0.82 | 1.04 | 1.36 | 1.85 | 1.36 | 1.24 | 1.19 | 1.15 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 | 1.07 | 1.07 | 1.06 |
| -1.00 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.92 | 0.90 | 0.89 | 0.87 | 0.84 | 0.80 | 0.70 | 0.59 | 0.87 | 1.08 | 1.45 | 1.53 | 1.28 | 1.20 | 1.16 | 1.13 | 1.11 | 1.10 | 1.08 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 |
| -1.10 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.90 | 0.88 | 0.85 | 0.80 | 0.70 | 0.67 | 0.91 | 1.12 | 1.61 | 1.39 | 1.23 | 1.17 | 1.13 | 1.11 | 1.09 | 1.08 | 1.07 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 |
| -1.20 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.86 | 0.81 | 0.70 | 0.73 | 0.95 | 1.17 | 2.35 | 1.30 | 1.19 | 1.14 | 1.12 | 1.10 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 | 1.05 | 1.04 |
| -1.30 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.87 | 0.82 | 0.70 | 0.78 | 0.98 | 1.21 | 1.60 | 1.24 | 1.16 | 1.12 | 1.10 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 |
| -1.40 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.90 | 0.87 | 0.83 | 0.69 | 0.82 | 1.01 | 1.26 | 1.40 | 1.20 | 1.14 | 1.11 | 1.09 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 |
| -1.50 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.91 | 0.88 | 0.83 | 0.67 | 0.85 | 1.04 | 1.32 | 1.30 | 1.17 | 1.12 | 1.10 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 | 1.03 |
| -1.60 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.91 | 0.88 | 0.83 | 0.63 | 0.88 | 1.06 | 1.42 | 1.24 | 1.15 | 1.11 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 |
| -1.70 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.91 | 0.89 | 0.84 | 0.52 | 0.91 | 1.09 | 1.76 | 1.20 | 1.13 | 1.09 | 1.08 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.03 |
| -1.80 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.92 | 0.89 | 0.84 | 0.63 | 0.93 | 1.11 | 1.47 | 1.17 | 1.11 | 1.08 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 |
| -1.90 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.92 | 0.90 | 0.84 | 0.71 | 0.95 | 1.14 | 1.32 | 1.15 | 1.10 | 1.08 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 |
| -2.00 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.90 | 0.84 | 0.76 | 0.97 | 1.17 | 1.25 | 1.13 | 1.09 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 |
| -2.10 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.90 | 0.85 | 0.80 | 0.99 | 1.20 | 1.20 | 1.11 | 1.08 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| -2.20 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.93 | 0.91 | 0.85 | 0.83 | 1.01 | 1.25 | 1.17 | 1.10 | 1.07 | 1.06 | 1.05 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| -2.30 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.93 | 0.91 | 0.85 | 0.85 | 1.02 | 1.32 | 1.15 | 1.09 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| -2.40 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.95 | 0.94 | 0.91 | 0.85 | 0.88 | 1.04 | 1.60 | 1.13 | 1.08 | 1.06 | 1.05 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 |
| -2.50 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.95 | 0.94 | 0.91 | 0.84 | 0.89 | 1.05 | 1.34 | 1.11 | 1.07 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 |
| -2.60 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.96 | 0.95 | 0.94 | 0.92 | 0.84 | 0.91 | 1.07 | 1.24 | 1.10 | 1.07 | 1.05 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 |
| -2.70 | 0.99 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.96 | 0.96 | 0.94 | 0.92 | 0.84 | 0.92 | 1.08 | 1.20 | 1.09 | 1.06 | 1.05 | 1.04 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 |
| -2.80 | 0.99 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.94 | 0.92 | 0.84 | 0.94 | 1.09 | 1.16 | 1.08 | 1.06 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 |
| -2.90 | 0.99 | 0.99 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.96 | 0.95 | 0.92 | 0.83 | 0.95 | 1.11 | 1.14 | 1.07 | 1.05 | 1.04 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |
| -3.00 | 0.99 | 0.99 | 0.99 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.96 | 0.95 | 0.92 | 0.82 | 0.96 | 1.13 | 1.12 | 1.07 | 1.05 | 1.04 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 | 1.01 |

🟧 $AREFF_{CH|\overline{CH}} \geq 2$ 🟦 $1 \leq AREFF_{CH|\overline{CH}} < 2$ 🟦 $0.75 \leq AREFF_{CH|\overline{CH}} < 1$ ⬜ $AREFF_{CH|\overline{CH}} < 0.75$

Figure 1: $AREFF_{CH|\overline{CH}}$, in the $(\xi, \rho)$-plane.

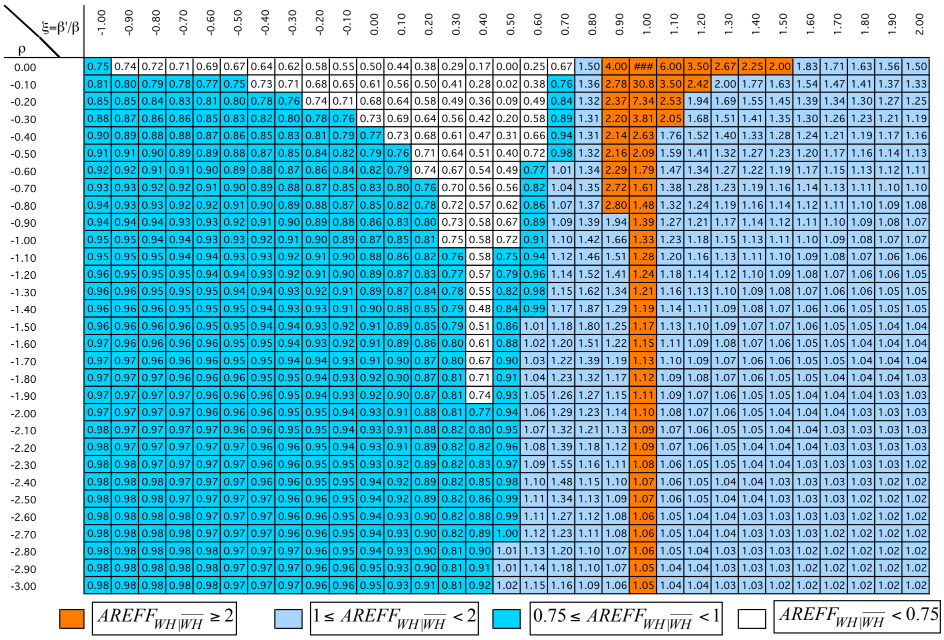
From these figures it is possible to see that, as expected, there is not a big difference between the relative behaviour between $UH$ and $\overline{UH}$ for $UH = CH, ML$ and $CH$, but their simultaneous use will for sure enable us to better estimate $\gamma$, the primary parameter of extreme events.



| ξ=β'/β ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.75 | 0.74 | 0.72 | 0.71 | 0.69 | 0.67 | 0.64 | 0.62 | 0.58 | 0.55 | 0.50 | 0.44 | 0.38 | 0.29 | 0.17 | 0.00 | 0.25 | 0.67 | 1.50 | 4.00 | ### | 6.00 | 3.50 | 2.67 | 2.25 | 2.00 | 1.83 | 1.71 | 1.63 | 1.56 | 1.50 |
| -0.10 | 0.81 | 0.80 | 0.79 | 0.78 | 0.77 | 0.75 | 0.73 | 0.71 | 0.68 | 0.65 | 0.61 | 0.56 | 0.50 | 0.41 | 0.28 | 0.00 | 0.37 | 0.75 | 1.34 | 2.69 | ### | 3.60 | 2.45 | 2.01 | 1.78 | 1.64 | 1.54 | 1.47 | 1.41 | 1.37 | 1.34 |
| -0.20 | 0.85 | 0.84 | 0.83 | 0.82 | 0.81 | 0.80 | 0.78 | 0.76 | 0.74 | 0.71 | 0.68 | 0.64 | 0.58 | 0.50 | 0.37 | 0.00 | 0.46 | 0.80 | 1.25 | 2.16 | ### | 2.71 | 2.01 | 1.72 | 1.57 | 1.47 | 1.40 | 1.35 | 1.31 | 1.28 | 1.25 |
| -0.30 | 0.88 | 0.87 | 0.86 | 0.85 | 0.84 | 0.83 | 0.82 | 0.80 | 0.78 | 0.76 | 0.73 | 0.69 | 0.64 | 0.57 | 0.44 | 0.00 | 0.53 | 0.83 | 1.20 | 1.88 | ### | 2.26 | 1.77 | 1.56 | 1.45 | 1.37 | 1.32 | 1.28 | 1.25 | 1.22 | 1.20 |
| -0.40 | 0.90 | 0.89 | 0.88 | 0.87 | 0.87 | 0.86 | 0.84 | 0.83 | 0.81 | 0.79 | 0.77 | 0.73 | 0.69 | 0.62 | 0.50 | 0.00 | 0.59 | 0.86 | 1.17 | 1.70 | ### | 1.99 | 1.62 | 1.46 | 1.37 | 1.31 | 1.26 | 1.23 | 1.21 | 1.19 | 1.17 |
| -0.50 | 0.91 | 0.90 | 0.90 | 0.89 | 0.88 | 0.87 | 0.86 | 0.85 | 0.84 | 0.82 | 0.79 | 0.76 | 0.72 | 0.66 | 0.55 | 0.00 | 0.63 | 0.87 | 1.14 | 1.59 | ### | 1.82 | 1.52 | 1.39 | 1.31 | 1.26 | 1.22 | 1.20 | 1.18 | 1.16 | 1.14 |
| -0.60 | 0.92 | 0.91 | 0.91 | 0.90 | 0.90 | 0.89 | 0.88 | 0.87 | 0.85 | 0.84 | 0.82 | 0.79 | 0.75 | 0.69 | 0.59 | 0.00 | 0.67 | 0.89 | 1.13 | 1.50 | ### | 1.69 | 1.45 | 1.33 | 1.27 | 1.23 | 1.20 | 1.17 | 1.15 | 1.14 | 1.13 |
| -0.70 | 0.93 | 0.92 | 0.92 | 0.91 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.85 | 0.83 | 0.81 | 0.77 | 0.72 | 0.62 | 0.00 | 0.69 | 0.90 | 1.11 | 1.44 | ### | 1.60 | 1.39 | 1.29 | 1.24 | 1.20 | 1.17 | 1.15 | 1.14 | 1.12 | 1.11 |
| -0.80 | 0.93 | 0.93 | 0.93 | 0.92 | 0.91 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.85 | 0.82 | 0.79 | 0.74 | 0.65 | 0.00 | 0.72 | 0.91 | 1.10 | 1.39 | ### | 1.53 | 1.35 | 1.26 | 1.21 | 1.18 | 1.16 | 1.14 | 1.12 | 1.11 | 1.10 |
| -0.90 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.86 | 0.84 | 0.81 | 0.76 | 0.68 | 0.00 | 0.74 | 0.92 | 1.09 | 1.35 | ### | 1.48 | 1.31 | 1.24 | 1.19 | 1.16 | 1.14 | 1.12 | 1.11 | 1.10 | 1.09 |
| -1.00 | 0.94 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.89 | 0.87 | 0.85 | 0.82 | 0.78 | 0.70 | 0.00 | 0.76 | 0.92 | 1.08 | 1.32 | 944 | 1.43 | 1.28 | 1.22 | 1.18 | 1.15 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 |
| -1.10 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.91 | 0.91 | 0.89 | 0.88 | 0.86 | 0.83 | 0.79 | 0.72 | 0.00 | 0.77 | 0.93 | 1.08 | 1.29 | 568 | 1.39 | 1.26 | 1.20 | 1.16 | 1.14 | 1.12 | 1.10 | 1.09 | 1.09 | 1.08 |
| -1.20 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.87 | 0.84 | 0.81 | 0.73 | 0.00 | 0.79 | 0.93 | 1.07 | 1.27 | 367 | 1.36 | 1.24 | 1.18 | 1.15 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 | 1.07 |
| -1.30 | 0.95 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.92 | 0.91 | 0.89 | 0.88 | 0.85 | 0.82 | 0.75 | 0.00 | 0.80 | 0.94 | 1.07 | 1.25 | 251 | 1.34 | 1.22 | 1.17 | 1.14 | 1.12 | 1.10 | 1.09 | 1.08 | 1.07 | 1.07 |
| -1.40 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.90 | 0.88 | 0.86 | 0.83 | 0.76 | 0.01 | 0.81 | 0.94 | 1.06 | 1.23 | 179 | 1.31 | 1.21 | 1.16 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 | 1.07 | 1.06 |
| -1.50 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.91 | 0.89 | 0.87 | 0.84 | 0.77 | 0.01 | 0.82 | 0.94 | 1.06 | 1.22 | 133 | 1.29 | 1.20 | 1.15 | 1.12 | 1.10 | 1.09 | 1.08 | 1.07 | 1.07 | 1.06 |
| -1.60 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.90 | 0.88 | 0.84 | 0.78 | 0.01 | 0.83 | 0.95 | 1.06 | 1.21 | 102 | 1.27 | 1.18 | 1.14 | 1.12 | 1.10 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 |
| -1.70 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.91 | 0.90 | 0.88 | 0.85 | 0.79 | 0.01 | 0.84 | 0.95 | 1.05 | 1.19 | 80.7 | 1.26 | 1.17 | 1.13 | 1.11 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 |
| -1.80 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.86 | 0.80 | 0.02 | 0.84 | 0.95 | 1.05 | 1.18 | 65.2 | 1.24 | 1.17 | 1.13 | 1.10 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 |
| -1.90 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.86 | 0.81 | 0.02 | 0.85 | 0.95 | 1.05 | 1.17 | 53.7 | 1.23 | 1.16 | 1.12 | 1.10 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 |
| -2.00 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.93 | 0.91 | 0.90 | 0.87 | 0.82 | 0.02 | 0.86 | 0.96 | 1.05 | 1.17 | 45.0 | 1.22 | 1.15 | 1.12 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 |
| -2.10 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.90 | 0.88 | 0.83 | 0.03 | 0.86 | 0.96 | 1.04 | 1.16 | 38.2 | 1.21 | 1.14 | 1.11 | 1.09 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 |
| -2.20 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.90 | 0.88 | 0.83 | 0.03 | 0.87 | 0.96 | 1.04 | 1.15 | 33.0 | 1.20 | 1.14 | 1.11 | 1.09 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 | 1.04 |
| -2.30 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.91 | 0.88 | 0.84 | 0.03 | 0.87 | 0.96 | 1.04 | 1.15 | 28.7 | 1.19 | 1.13 | 1.10 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 |
| -2.40 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.94 | 0.94 | 0.93 | 0.91 | 0.89 | 0.84 | 0.04 | 0.88 | 0.96 | 1.04 | 1.14 | 25.3 | 1.18 | 1.13 | 1.10 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 |
| -2.50 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.91 | 0.89 | 0.85 | 0.04 | 0.88 | 0.96 | 1.04 | 1.13 | 22.5 | 1.18 | 1.12 | 1.09 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 |
| -2.60 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.90 | 0.85 | 0.05 | 0.89 | 0.97 | 1.04 | 1.13 | 20.2 | 1.17 | 1.12 | 1.09 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 |
| -2.70 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.92 | 0.90 | 0.86 | 0.05 | 0.89 | 0.97 | 1.03 | 1.12 | 18.2 | 1.16 | 1.11 | 1.09 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 |
| -2.80 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.94 | 0.94 | 0.92 | 0.90 | 0.86 | 0.06 | 0.89 | 0.97 | 1.03 | 1.12 | 16.6 | 1.16 | 1.11 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 |
| -2.90 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.91 | 0.87 | 0.07 | 0.90 | 0.97 | 1.03 | 1.12 | 15.2 | 1.15 | 1.10 | 1.08 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 |
| -3.00 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.91 | 0.87 | 0.07 | 0.90 | 0.97 | 1.03 | 1.11 | 13.9 | 1.15 | 1.10 | 1.08 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 |

🟧 $AREFF_{ML|\overline{ML}} \geq 2$ ▨ $1 \leq AREFF_{ML|\overline{ML}} < 2$ 🟦 $0.75 \leq AREFF_{ML|\overline{ML}} < 1$ ☐ $AREFF_{ML|\overline{ML}} < 0.75$

Figure 2: $AREFF_{ML|\overline{ML}}$, in the $(\xi, \rho)$-plane.

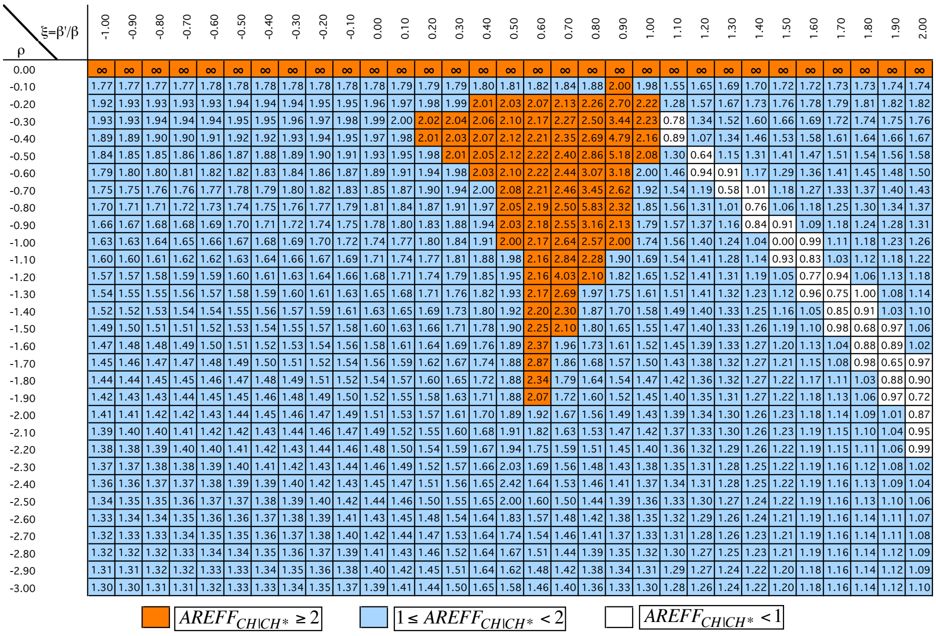


| ξ=β'/β ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 0.75 | 0.74 | 0.72 | 0.71 | 0.69 | 0.67 | 0.64 | 0.62 | 0.58 | 0.55 | 0.50 | 0.44 | 0.38 | 0.29 | 0.17 | 0.00 | 0.25 | 0.67 | 1.50 | 4.00 | ### | 6.00 | 3.50 | 2.67 | 2.25 | 2.00 | 1.83 | 1.71 | 1.63 | 1.56 | 1.50 |
| -0.10 | 0.81 | 0.80 | 0.79 | 0.78 | 0.77 | 0.75 | 0.73 | 0.71 | 0.68 | 0.65 | 0.61 | 0.56 | 0.50 | 0.41 | 0.28 | 0.02 | 0.38 | 0.76 | 1.36 | 2.78 | 30.8 | 3.50 | 2.42 | 2.00 | 1.77 | 1.63 | 1.54 | 1.47 | 1.41 | 1.37 | 1.33 |
| -0.20 | 0.85 | 0.85 | 0.84 | 0.83 | 0.81 | 0.80 | 0.78 | 0.76 | 0.74 | 0.71 | 0.68 | 0.64 | 0.58 | 0.49 | 0.36 | 0.09 | 0.49 | 0.84 | 1.32 | 2.37 | 7.34 | 2.53 | 1.94 | 1.69 | 1.55 | 1.45 | 1.39 | 1.34 | 1.30 | 1.27 | 1.25 |
| -0.30 | 0.88 | 0.87 | 0.86 | 0.86 | 0.85 | 0.83 | 0.82 | 0.80 | 0.78 | 0.76 | 0.73 | 0.69 | 0.64 | 0.56 | 0.42 | 0.20 | 0.58 | 0.89 | 1.31 | 2.20 | 3.81 | 2.05 | 1.68 | 1.51 | 1.41 | 1.35 | 1.30 | 1.26 | 1.23 | 1.21 | 1.19 |
| -0.40 | 0.90 | 0.89 | 0.88 | 0.88 | 0.87 | 0.86 | 0.85 | 0.83 | 0.81 | 0.79 | 0.77 | 0.73 | 0.68 | 0.61 | 0.47 | 0.31 | 0.66 | 0.94 | 1.31 | 2.14 | 2.63 | 1.76 | 1.52 | 1.40 | 1.33 | 1.28 | 1.24 | 1.21 | 1.19 | 1.17 | 1.16 |
| -0.50 | 0.91 | 0.91 | 0.90 | 0.89 | 0.89 | 0.88 | 0.87 | 0.85 | 0.84 | 0.82 | 0.79 | 0.76 | 0.71 | 0.64 | 0.51 | 0.40 | 0.72 | 0.98 | 1.32 | 2.16 | 2.09 | 1.59 | 1.41 | 1.32 | 1.27 | 1.23 | 1.20 | 1.17 | 1.16 | 1.14 | 1.13 |
| -0.60 | 0.92 | 0.92 | 0.91 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.86 | 0.84 | 0.82 | 0.79 | 0.74 | 0.67 | 0.54 | 0.49 | 0.77 | 1.01 | 1.34 | 2.29 | 1.79 | 1.47 | 1.34 | 1.27 | 1.22 | 1.19 | 1.17 | 1.15 | 1.13 | 1.12 | 1.11 |
| -0.70 | 0.93 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.85 | 0.83 | 0.80 | 0.76 | 0.70 | 0.56 | 0.56 | 0.82 | 1.04 | 1.35 | 2.72 | 1.61 | 1.38 | 1.28 | 1.23 | 1.19 | 1.16 | 1.14 | 1.13 | 1.11 | 1.10 | 1.10 |
| -0.80 | 0.94 | 0.93 | 0.93 | 0.92 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.87 | 0.85 | 0.82 | 0.78 | 0.72 | 0.57 | 0.62 | 0.86 | 1.07 | 1.37 | 2.80 | 1.48 | 1.32 | 1.24 | 1.19 | 1.16 | 1.14 | 1.12 | 1.11 | 1.10 | 1.09 | 1.08 |
| -0.90 | 0.94 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.88 | 0.86 | 0.83 | 0.80 | 0.73 | 0.58 | 0.67 | 0.89 | 1.09 | 1.39 | 1.94 | 1.39 | 1.27 | 1.21 | 1.17 | 1.14 | 1.12 | 1.11 | 1.10 | 1.09 | 1.08 | 1.07 |
| -1.00 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.87 | 0.85 | 0.81 | 0.75 | 0.58 | 0.72 | 0.91 | 1.10 | 1.42 | 1.66 | 1.33 | 1.23 | 1.18 | 1.15 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 | 1.07 | 1.07 |
| -1.10 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.93 | 0.92 | 0.91 | 0.90 | 0.88 | 0.86 | 0.82 | 0.76 | 0.58 | 0.75 | 0.94 | 1.12 | 1.46 | 1.51 | 1.28 | 1.20 | 1.16 | 1.13 | 1.11 | 1.10 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 |
| -1.20 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.90 | 0.89 | 0.87 | 0.83 | 0.77 | 0.57 | 0.79 | 0.96 | 1.14 | 1.52 | 1.41 | 1.24 | 1.18 | 1.14 | 1.12 | 1.10 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 |
| -1.30 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.87 | 0.84 | 0.78 | 0.55 | 0.82 | 0.98 | 1.15 | 1.62 | 1.34 | 1.21 | 1.16 | 1.13 | 1.10 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 |
| -1.40 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.95 | 0.94 | 0.93 | 0.93 | 0.91 | 0.90 | 0.88 | 0.85 | 0.79 | 0.48 | 0.84 | 0.99 | 1.17 | 1.87 | 1.29 | 1.19 | 1.14 | 1.11 | 1.09 | 1.08 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 | 1.05 |
| -1.50 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.85 | 0.79 | 0.51 | 0.86 | 1.01 | 1.18 | 1.80 | 1.25 | 1.17 | 1.13 | 1.10 | 1.09 | 1.07 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 |
| -1.60 | 0.97 | 0.96 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.91 | 0.89 | 0.86 | 0.80 | 0.61 | 0.88 | 1.02 | 1.20 | 1.51 | 1.22 | 1.15 | 1.11 | 1.09 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 |
| -1.70 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.94 | 0.93 | 0.91 | 0.90 | 0.87 | 0.80 | 0.67 | 0.90 | 1.03 | 1.22 | 1.39 | 1.19 | 1.13 | 1.10 | 1.09 | 1.07 | 1.06 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 |
| -1.80 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.92 | 0.90 | 0.87 | 0.81 | 0.71 | 0.91 | 1.04 | 1.23 | 1.32 | 1.17 | 1.12 | 1.09 | 1.08 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 |
| -1.90 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.92 | 0.90 | 0.87 | 0.81 | 0.74 | 0.93 | 1.05 | 1.26 | 1.27 | 1.15 | 1.11 | 1.09 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 | 1.03 |
| -2.00 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.91 | 0.88 | 0.81 | 0.77 | 0.94 | 1.06 | 1.29 | 1.23 | 1.14 | 1.10 | 1.08 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 |
| -2.10 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.93 | 0.91 | 0.88 | 0.82 | 0.80 | 0.95 | 1.07 | 1.32 | 1.21 | 1.13 | 1.09 | 1.07 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 |
| -2.20 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.91 | 0.89 | 0.82 | 0.82 | 0.96 | 1.08 | 1.39 | 1.18 | 1.12 | 1.09 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 |
| -2.30 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.93 | 0.92 | 0.89 | 0.82 | 0.83 | 0.97 | 1.09 | 1.55 | 1.16 | 1.11 | 1.08 | 1.06 | 1.05 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 |
| -2.40 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.95 | 0.94 | 0.92 | 0.89 | 0.82 | 0.85 | 0.98 | 1.10 | 1.48 | 1.15 | 1.10 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 |
| -2.50 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.92 | 0.89 | 0.82 | 0.86 | 0.99 | 1.11 | 1.34 | 1.13 | 1.09 | 1.07 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 |
| -2.60 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.90 | 0.82 | 0.88 | 0.99 | 1.11 | 1.27 | 1.12 | 1.08 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 |
| -2.70 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.94 | 0.93 | 0.90 | 0.82 | 0.89 | 1.00 | 1.12 | 1.23 | 1.11 | 1.08 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 |
| -2.80 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.95 | 0.94 | 0.93 | 0.90 | 0.81 | 0.90 | 1.01 | 1.13 | 1.20 | 1.10 | 1.07 | 1.06 | 1.05 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| -2.90 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.93 | 0.90 | 0.81 | 0.91 | 1.01 | 1.14 | 1.18 | 1.10 | 1.07 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |
| -3.00 | 0.98 | 0.98 | 0.98 | 0.98 | 0.98 | 0.97 | 0.97 | 0.97 | 0.96 | 0.96 | 0.95 | 0.93 | 0.91 | 0.81 | 0.92 | 1.02 | 1.15 | 1.16 | 1.09 | 1.06 | 1.05 | 1.04 | 1.04 | 1.03 | 1.03 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 | 1.02 |

🟧 $AREFF_{WH|\overline{WH}} \geq 2$ ▨ $1 \leq AREFF_{WH|\overline{WH}} < 2$ 🟦 $0.75 \leq AREFF_{WH|\overline{WH}} < 1$ ☐ $AREFF_{WH|\overline{WH}} < 0.75$

Figure 3: $AREFF_{WH|\overline{WH}}$, in the $(\xi, \rho)$-plane.

As already proved in Proposition 2.1 of Gomes *et al.* (2007a), $ML$ beats $ML^*$ asymptotically at optimal levels for all $\beta \neq \beta'$ and $\rho < 0$. Asymptotically and at optimal levels, $WH$ also beats $WH^*$ over almost all $(\xi, \rho)$-plane. A similar comment applies to the equivalent behaviour of $CH$ relatively to $CH^*$, as can be seen from Figure 4, illustrative of the comparative behaviour of the statistics $UH$ and $UH^*$.



| ξ=β'/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 1.77 | 1.77 | 1.77 | 1.77 | 1.78 | 1.78 | 1.78 | 1.78 | 1.78 | 1.78 | 1.78 | 1.79 | 1.79 | 1.79 | 1.80 | 1.81 | 1.82 | 1.84 | 1.88 | 2.00 | 1.98 | 1.55 | 1.65 | 1.69 | 1.70 | 1.72 | 1.72 | 1.73 | 1.73 | 1.74 | 1.74 |
| -0.20 | 1.92 | 1.93 | 1.93 | 1.93 | 1.93 | 1.94 | 1.94 | 1.94 | 1.95 | 1.95 | 1.96 | 1.97 | 1.98 | 1.99 | 2.01 | 2.03 | 2.07 | 2.13 | 2.26 | 2.70 | 2.22 | 1.28 | 1.57 | 1.67 | 1.73 | 1.76 | 1.78 | 1.79 | 1.81 | 1.82 | 1.82 |
| -0.30 | 1.93 | 1.93 | 1.94 | 1.94 | 1.94 | 1.95 | 1.95 | 1.96 | 1.97 | 1.98 | 1.99 | 2.00 | 2.02 | 2.04 | 2.06 | 2.10 | 2.17 | 2.27 | 2.50 | 3.44 | 2.23 | 0.78 | 1.34 | 1.52 | 1.60 | 1.66 | 1.69 | 1.72 | 1.74 | 1.75 | 1.76 |
| -0.40 | 1.89 | 1.89 | 1.90 | 1.90 | 1.91 | 1.92 | 1.92 | 1.93 | 1.94 | 1.95 | 1.97 | 1.98 | 2.01 | 2.03 | 2.07 | 2.12 | 2.21 | 2.35 | 2.69 | 4.79 | 2.16 | 0.89 | 1.07 | 1.34 | 1.46 | 1.53 | 1.58 | 1.61 | 1.64 | 1.66 | 1.67 |
| -0.50 | 1.84 | 1.85 | 1.85 | 1.86 | 1.86 | 1.87 | 1.88 | 1.89 | 1.90 | 1.91 | 1.93 | 1.95 | 1.98 | 2.01 | 2.05 | 2.12 | 2.22 | 2.40 | 2.86 | 5.18 | 2.08 | 1.30 | 0.64 | 1.15 | 1.31 | 1.41 | 1.47 | 1.51 | 1.54 | 1.56 | 1.58 |
| -0.60 | 1.79 | 1.80 | 1.80 | 1.81 | 1.82 | 1.82 | 1.83 | 1.84 | 1.86 | 1.87 | 1.89 | 1.91 | 1.94 | 1.98 | 2.03 | 2.10 | 2.22 | 2.44 | 3.07 | 3.18 | 2.00 | 1.46 | 0.94 | 0.91 | 1.17 | 1.29 | 1.36 | 1.41 | 1.45 | 1.48 | 1.50 |
| -0.70 | 1.75 | 1.75 | 1.76 | 1.76 | 1.77 | 1.78 | 1.79 | 1.80 | 1.82 | 1.83 | 1.85 | 1.87 | 1.90 | 1.94 | 2.00 | 2.08 | 2.21 | 2.46 | 3.45 | 2.62 | 1.92 | 1.54 | 1.19 | 0.58 | 1.01 | 1.18 | 1.27 | 1.33 | 1.37 | 1.40 | 1.43 |
| -0.80 | 1.70 | 1.71 | 1.71 | 1.72 | 1.73 | 1.74 | 1.75 | 1.76 | 1.77 | 1.79 | 1.81 | 1.84 | 1.87 | 1.91 | 1.97 | 2.05 | 2.19 | 2.50 | 5.83 | 2.32 | 1.85 | 1.56 | 1.31 | 1.01 | 0.76 | 1.06 | 1.18 | 1.25 | 1.30 | 1.34 | 1.37 |
| -0.90 | 1.66 | 1.67 | 1.68 | 1.68 | 1.69 | 1.70 | 1.71 | 1.72 | 1.74 | 1.75 | 1.78 | 1.80 | 1.83 | 1.88 | 1.94 | 2.03 | 2.18 | 2.55 | 3.16 | 2.13 | 1.79 | 1.57 | 1.37 | 1.16 | 0.84 | 0.91 | 1.09 | 1.18 | 1.24 | 1.28 | 1.31 |
| -1.00 | 1.63 | 1.63 | 1.64 | 1.65 | 1.66 | 1.67 | 1.68 | 1.69 | 1.70 | 1.72 | 1.74 | 1.77 | 1.80 | 1.84 | 1.91 | 2.00 | 2.17 | 2.64 | 2.57 | 2.00 | 1.74 | 1.56 | 1.40 | 1.24 | 1.04 | 0.00 | 0.99 | 1.11 | 1.18 | 1.23 | 1.26 |
| -1.10 | 1.60 | 1.60 | 1.61 | 1.62 | 1.62 | 1.63 | 1.64 | 1.66 | 1.67 | 1.69 | 1.71 | 1.74 | 1.77 | 1.81 | 1.88 | 1.98 | 2.16 | 2.84 | 2.28 | 1.90 | 1.69 | 1.54 | 1.41 | 1.28 | 1.14 | 0.93 | 0.83 | 1.03 | 1.12 | 1.18 | 1.22 |
| -1.20 | 1.57 | 1.57 | 1.58 | 1.59 | 1.59 | 1.60 | 1.61 | 1.63 | 1.64 | 1.66 | 1.68 | 1.71 | 1.74 | 1.79 | 1.85 | 1.95 | 2.16 | 4.03 | 2.10 | 1.82 | 1.65 | 1.52 | 1.41 | 1.31 | 1.19 | 1.05 | 0.77 | 0.94 | 1.06 | 1.13 | 1.18 |
| -1.30 | 1.54 | 1.55 | 1.55 | 1.56 | 1.57 | 1.58 | 1.59 | 1.60 | 1.61 | 1.63 | 1.65 | 1.68 | 1.71 | 1.76 | 1.82 | 1.93 | 2.17 | 2.69 | 1.97 | 1.75 | 1.61 | 1.51 | 1.41 | 1.32 | 1.23 | 1.12 | 0.96 | 0.75 | 1.00 | 1.08 | 1.14 |
| -1.40 | 1.52 | 1.52 | 1.53 | 1.54 | 1.54 | 1.55 | 1.56 | 1.57 | 1.59 | 1.61 | 1.63 | 1.65 | 1.69 | 1.73 | 1.80 | 1.92 | 2.20 | 2.30 | 1.87 | 1.70 | 1.58 | 1.49 | 1.40 | 1.33 | 1.25 | 1.16 | 1.05 | 0.85 | 0.91 | 1.03 | 1.10 |
| -1.50 | 1.49 | 1.50 | 1.51 | 1.51 | 1.52 | 1.53 | 1.54 | 1.55 | 1.57 | 1.58 | 1.60 | 1.63 | 1.66 | 1.71 | 1.78 | 1.90 | 2.25 | 2.10 | 1.80 | 1.65 | 1.55 | 1.47 | 1.40 | 1.33 | 1.26 | 1.19 | 1.10 | 0.98 | 0.68 | 0.97 | 1.06 |
| -1.60 | 1.47 | 1.48 | 1.48 | 1.49 | 1.50 | 1.51 | 1.52 | 1.53 | 1.54 | 1.56 | 1.58 | 1.61 | 1.64 | 1.69 | 1.76 | 1.89 | 2.37 | 1.96 | 1.73 | 1.61 | 1.52 | 1.45 | 1.39 | 1.33 | 1.27 | 1.20 | 1.13 | 1.04 | 0.88 | 0.89 | 1.02 |
| -1.70 | 1.45 | 1.46 | 1.47 | 1.47 | 1.48 | 1.49 | 1.50 | 1.51 | 1.52 | 1.54 | 1.56 | 1.59 | 1.62 | 1.67 | 1.74 | 1.88 | 2.87 | 1.86 | 1.68 | 1.57 | 1.50 | 1.43 | 1.38 | 1.32 | 1.27 | 1.21 | 1.15 | 1.08 | 0.98 | 0.65 | 0.97 |
| -1.80 | 1.44 | 1.44 | 1.45 | 1.45 | 1.46 | 1.47 | 1.48 | 1.49 | 1.51 | 1.52 | 1.54 | 1.57 | 1.60 | 1.65 | 1.72 | 1.88 | 2.34 | 1.79 | 1.64 | 1.54 | 1.47 | 1.42 | 1.36 | 1.32 | 1.27 | 1.22 | 1.17 | 1.11 | 1.03 | 0.88 | 0.90 |
| -1.90 | 1.42 | 1.43 | 1.43 | 1.44 | 1.45 | 1.45 | 1.46 | 1.48 | 1.49 | 1.50 | 1.52 | 1.55 | 1.58 | 1.63 | 1.71 | 1.88 | 2.07 | 1.72 | 1.60 | 1.52 | 1.45 | 1.40 | 1.35 | 1.31 | 1.27 | 1.22 | 1.18 | 1.13 | 1.06 | 0.97 | 0.72 |
| -2.00 | 1.41 | 1.41 | 1.42 | 1.42 | 1.43 | 1.44 | 1.45 | 1.46 | 1.47 | 1.49 | 1.51 | 1.53 | 1.57 | 1.61 | 1.70 | 1.89 | 1.92 | 1.67 | 1.56 | 1.49 | 1.43 | 1.39 | 1.34 | 1.30 | 1.26 | 1.23 | 1.18 | 1.14 | 1.09 | 1.01 | 0.87 |
| -2.10 | 1.39 | 1.40 | 1.40 | 1.41 | 1.42 | 1.42 | 1.43 | 1.44 | 1.46 | 1.47 | 1.49 | 1.52 | 1.55 | 1.60 | 1.68 | 1.91 | 1.82 | 1.63 | 1.53 | 1.47 | 1.42 | 1.37 | 1.33 | 1.30 | 1.26 | 1.23 | 1.19 | 1.15 | 1.10 | 1.04 | 0.95 |
| -2.20 | 1.38 | 1.38 | 1.39 | 1.40 | 1.40 | 1.41 | 1.42 | 1.43 | 1.44 | 1.46 | 1.48 | 1.50 | 1.54 | 1.59 | 1.67 | 1.94 | 1.75 | 1.59 | 1.51 | 1.45 | 1.40 | 1.36 | 1.32 | 1.29 | 1.26 | 1.22 | 1.19 | 1.15 | 1.11 | 1.06 | 0.99 |
| -2.30 | 1.37 | 1.37 | 1.38 | 1.38 | 1.39 | 1.40 | 1.41 | 1.42 | 1.43 | 1.44 | 1.46 | 1.49 | 1.52 | 1.57 | 1.66 | 2.03 | 1.69 | 1.56 | 1.48 | 1.43 | 1.38 | 1.35 | 1.31 | 1.28 | 1.25 | 1.22 | 1.19 | 1.16 | 1.12 | 1.08 | 1.02 |
| -2.40 | 1.36 | 1.36 | 1.37 | 1.37 | 1.38 | 1.39 | 1.39 | 1.40 | 1.42 | 1.43 | 1.45 | 1.47 | 1.51 | 1.56 | 1.65 | 2.42 | 1.64 | 1.53 | 1.46 | 1.41 | 1.37 | 1.34 | 1.31 | 1.28 | 1.25 | 1.22 | 1.19 | 1.16 | 1.13 | 1.09 | 1.04 |
| -2.50 | 1.34 | 1.35 | 1.35 | 1.36 | 1.37 | 1.37 | 1.38 | 1.39 | 1.40 | 1.42 | 1.44 | 1.46 | 1.50 | 1.55 | 1.65 | 2.00 | 1.60 | 1.50 | 1.44 | 1.39 | 1.36 | 1.33 | 1.30 | 1.27 | 1.24 | 1.22 | 1.19 | 1.16 | 1.13 | 1.10 | 1.06 |
| -2.60 | 1.33 | 1.34 | 1.34 | 1.35 | 1.36 | 1.36 | 1.37 | 1.38 | 1.39 | 1.41 | 1.43 | 1.45 | 1.48 | 1.54 | 1.64 | 1.83 | 1.57 | 1.48 | 1.42 | 1.38 | 1.35 | 1.32 | 1.29 | 1.26 | 1.24 | 1.21 | 1.19 | 1.16 | 1.14 | 1.11 | 1.07 |
| -2.70 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.35 | 1.36 | 1.37 | 1.38 | 1.40 | 1.42 | 1.44 | 1.47 | 1.53 | 1.64 | 1.74 | 1.54 | 1.46 | 1.41 | 1.37 | 1.33 | 1.31 | 1.28 | 1.26 | 1.23 | 1.21 | 1.19 | 1.16 | 1.14 | 1.11 | 1.08 |
| -2.80 | 1.32 | 1.32 | 1.32 | 1.33 | 1.34 | 1.34 | 1.35 | 1.36 | 1.37 | 1.39 | 1.41 | 1.43 | 1.46 | 1.52 | 1.64 | 1.67 | 1.51 | 1.44 | 1.39 | 1.35 | 1.32 | 1.30 | 1.27 | 1.25 | 1.23 | 1.21 | 1.19 | 1.16 | 1.14 | 1.12 | 1.09 |
| -2.90 | 1.31 | 1.31 | 1.32 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.36 | 1.38 | 1.40 | 1.42 | 1.45 | 1.51 | 1.64 | 1.62 | 1.48 | 1.42 | 1.38 | 1.34 | 1.31 | 1.29 | 1.27 | 1.24 | 1.22 | 1.20 | 1.18 | 1.16 | 1.14 | 1.12 | 1.09 |
| -3.00 | 1.30 | 1.30 | 1.31 | 1.31 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.37 | 1.39 | 1.41 | 1.44 | 1.50 | 1.65 | 1.58 | 1.46 | 1.40 | 1.36 | 1.33 | 1.30 | 1.28 | 1.26 | 1.24 | 1.22 | 1.20 | 1.18 | 1.16 | 1.14 | 1.12 | 1.10 |

🟧 $AREFF_{CH|CH^*} \geq 2$ 🟦 $1 \leq AREFF_{CH|CH^*} < 2$ ☐ $AREFF_{CH|CH^*} < 1$

Figure 4: $AREFF_{CH|CH^*}$, in the $(\xi, \rho)$-plane.

Despite of the fact that we have $|b^*_{UH}| \leq |b_{UH}|$ for a large region of the $(\gamma, \rho)$-plane, as $\sigma_{UH} = \gamma < \sigma^*_{UH} = \gamma(1-\rho)/|\rho|$ for all $\rho$, the $UH$-statistics are able to outperform the $UH^*$-statistics at optimal levels, unless we are in a region of the $(\xi, \rho)$-plane around the line $b^*_{UH} = 0$. A similar comment applies to the relative behaviour of $\overline{UH}$ and $\overline{UH}^\star$, as well as to the comparative behaviour of $\overline{UH}$ and $UH^*$. This is indeed a point in favour of the $UH$ statistics. The lines in $(\xi, \rho)$-plane associated to null bias of the different statistics are presented in Figure 5.

The general behaviour of $UH$, $\overline{UH}$, $UH^*$ and $\overline{UH}^*$, again for $UH = CH, ML, WH$, is shown in Figures 6, 7 and 8, respectively, where we register the estimator with the highest efficiency among the ones considered, i.e., the one with maximum $AREFF$, or equivalently, minimum $LMSE$.

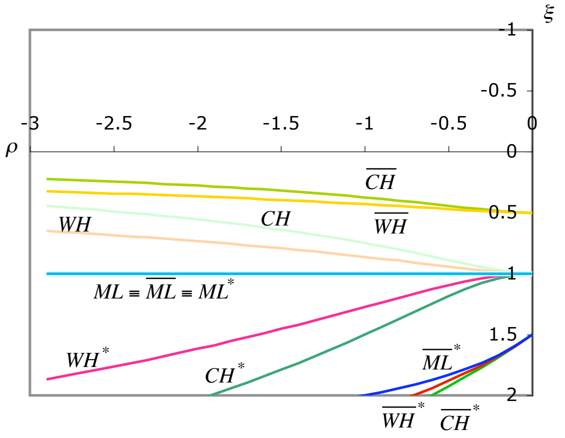
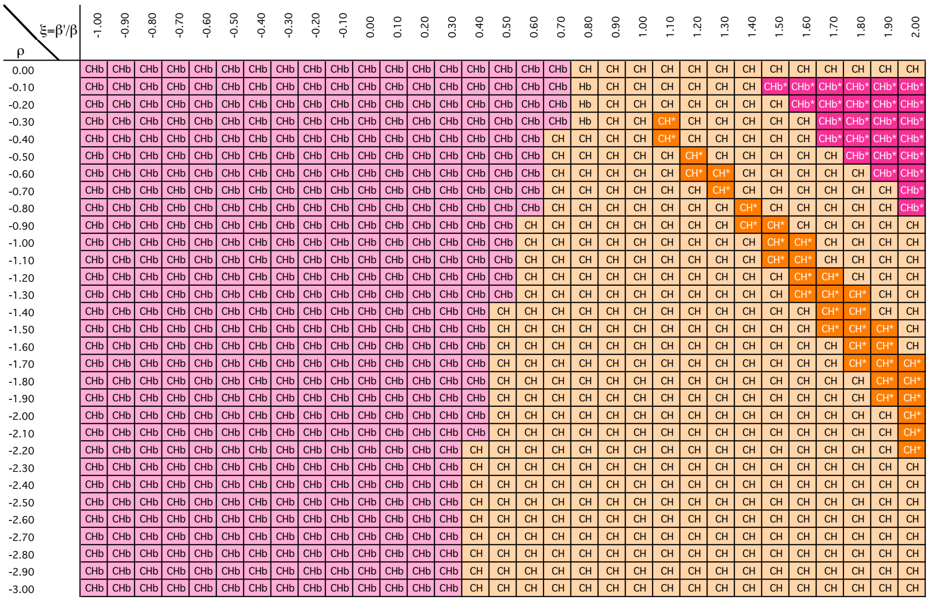


Figure 5: Lines where $b_{_{UH}} = 0$ and $b^*_{_{UH}} = 0$, $UH = CH, ML$ and $WH$.

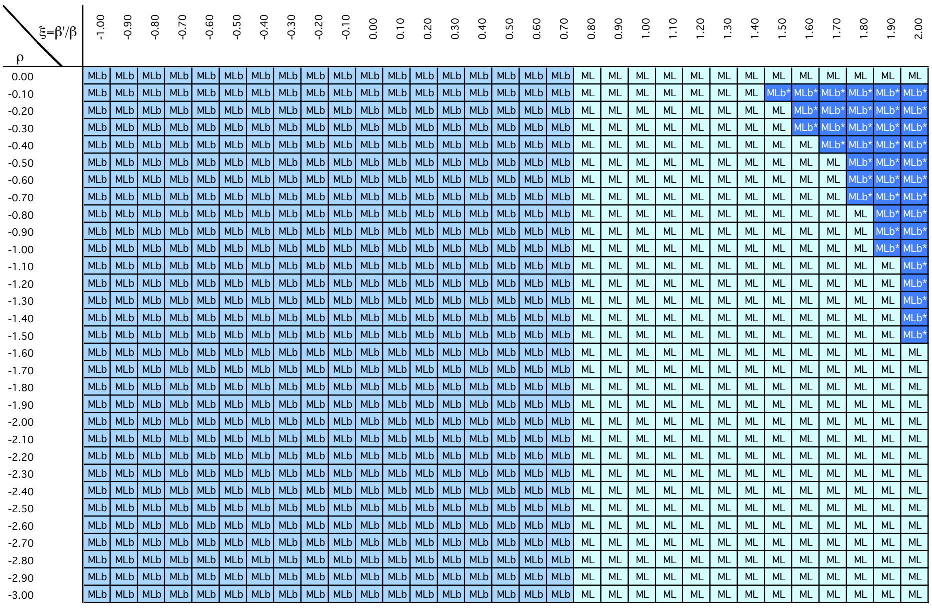


| ξ=β'/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -0.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | Hb | CH | CH | CH | CH | CH | CH | CHb* | CHb* | CHb* | CHb* | CHb* | CHb* |
| -0.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | Hb | CH | CH | CH | CH | CH | CH | CH | CHb* | CHb* | CHb* | CHb* | CHb* |
| -0.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | Hb | CH | CH | CH* | CH | CH | CH | CH | CH | CHb* | CHb* | CHb* | CHb* |
| -0.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH* | CH | CH | CH | CH | CH | CHb* | CHb* | CHb* | CHb* |
| -0.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH* | CH | CH | CH | CH | CH | CHb* | CHb* | CHb* |
| -0.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH* | CH* | CH | CH | CH | CH | CH | CHb* | CHb* |
| -0.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH* | CH | CH | CH | CH | CH | CH | CHb* |
| -0.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH* | CH | CH | CH | CH | CH | CHb* |
| -0.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH | CH | CH | CH | CH |
| -1.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH | CH | CH | CH |
| -1.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH | CH | CH | CH |
| -1.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH | CH | CH |
| -1.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH* | CH | CH |
| -1.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH | CH |
| -1.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH* | CH |
| -1.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH |
| -1.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* | CH* |
| -1.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* |
| -1.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* | CH* |
| -2.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* |
| -2.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* |
| -2.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH* |
| -2.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -2.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -2.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -2.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -2.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -2.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -2.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |
| -3.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH | CH |

Figure 6: Estimator with minimum LMSE among $CH$, $\overline{CH}$ (CHb), $CH^*$ and $\overline{CH}^*$ (CHb*).

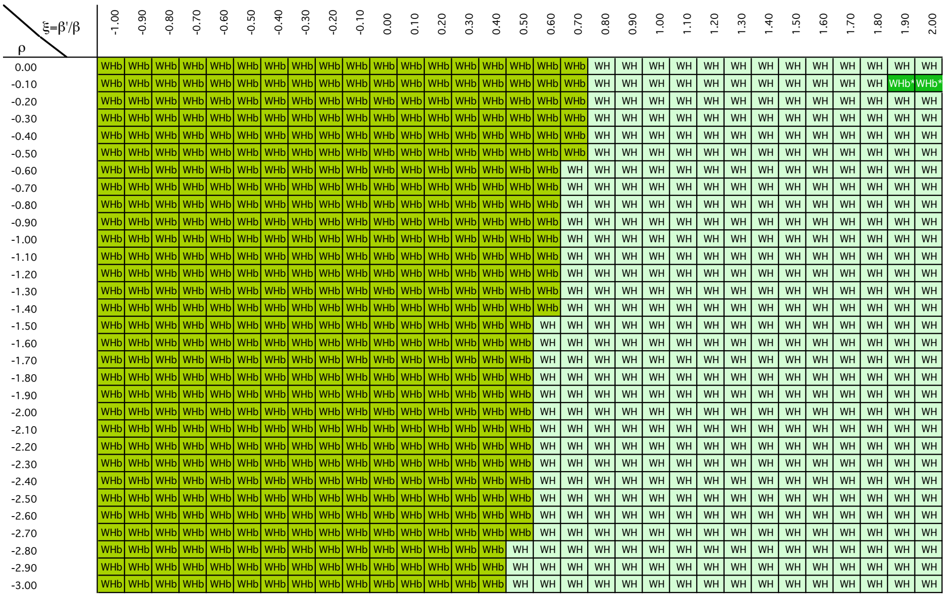
Finally, in Figure 9 we picture together all the $CH$ and $ML$ statistics and in Figure 10 we put together all the $CH$, the $ML$ and the $WH$ statistics.

As mentioned before, none of the estimators can always dominate the alternatives, but on the basis of initial estimates of $(\xi, \rho)$, the following figures can help us to choose the most adequate estimator. Estimators of $\rho$ have already been dealt with by several authors. Apart from the above



| ξ=β′/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -0.10 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* | MLb* | MLb* | MLb* |
| -0.20 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* | MLb* | MLb* |
| -0.30 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* | MLb* | MLb* |
| -0.40 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* | MLb* |
| -0.50 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* |
| -0.60 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* |
| -0.70 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* | MLb* |
| -0.80 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* |
| -0.90 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* |
| -1.00 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | MLb* |
| -1.10 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* |
| -1.20 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* |
| -1.30 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* |
| -1.40 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* |
| -1.50 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | MLb* |
| -1.60 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -1.70 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -1.80 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -1.90 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.00 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.10 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.20 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.30 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.40 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.50 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.60 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.70 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.80 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.90 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -3.00 | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |

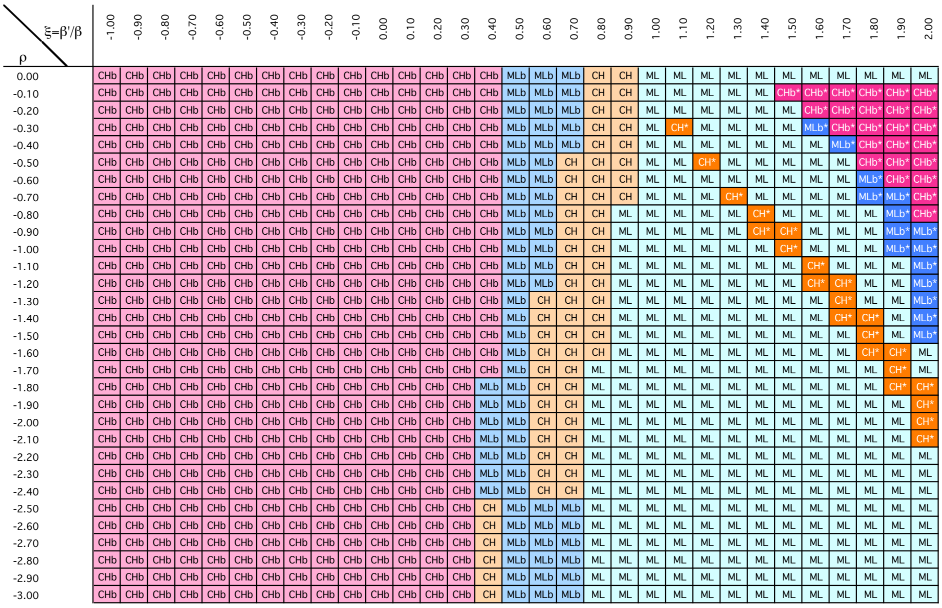
Figure 7: Estimator with minimum LMSE among $ML$, $\overline{ML}$ (MLb), $ML^*$ and $\overline{ML}^*$ (MLb*).



| ξ=β′/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.10 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WHb* | WHb* |
| -0.20 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.30 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.40 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.50 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.60 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.70 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.80 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -0.90 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.00 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.10 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.20 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.30 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.40 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.50 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.60 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.70 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.80 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -1.90 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.00 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.10 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.20 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.30 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.40 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.50 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.60 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.70 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.80 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -2.90 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |
| -3.00 | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WHb | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH | WH |

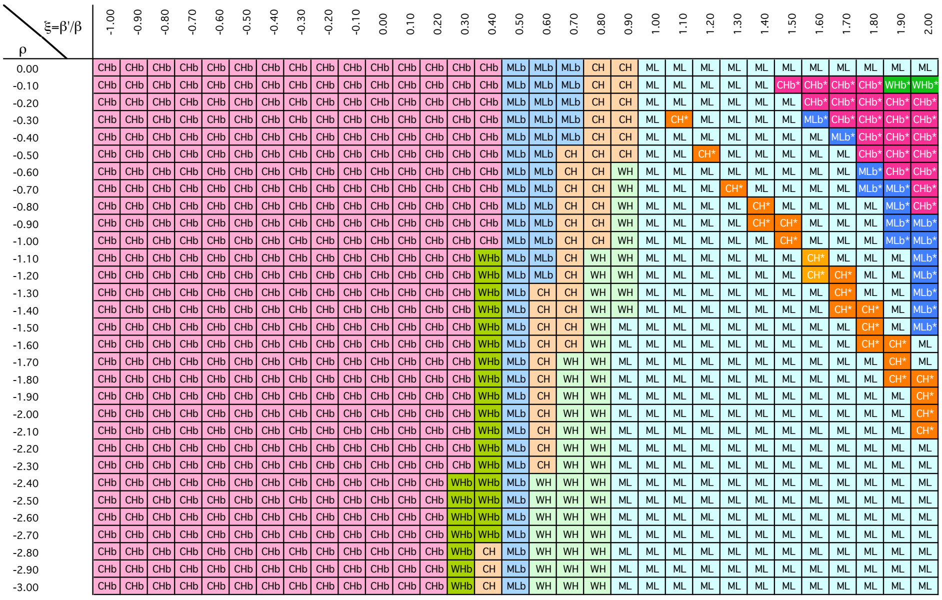
Figure 8: Estimator with minimum LMSE among $WH$, $\overline{WH}$ (WHb), $WH^*$ and $\overline{WH}^*$ (WHb*).

mentioned paper by Gomes *et al.* (2002) and Fraga Alves *et al.* (2003), we mention the recent papers by Ciuperca and Mercadier (2010) and Goegebeur *et al.* (2010). The estimation of $\xi$ is



| ξ=β'/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -0.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | CHb* | CHb* | CHb* | CHb* | CHb* | CHb* |
| -0.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | CHb* | CHb* | CHb* | CHb* | CHb* |
| -0.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | CH* | ML | ML | ML | ML | MLb* | CHb* | CHb* | CHb* | CHb* |
| -0.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | MLb* | CHb* | CHb* | CHb* |
| -0.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | CH | ML | ML | CH* | ML | ML | ML | ML | ML | CHb* | CHb* | CHb* |
| -0.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | CHb* | CHb* |
| -0.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | CH | ML | ML | ML | CH* | ML | ML | ML | ML | MLb* | MLb* | CHb* |
| -0.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | CH* | ML | ML | ML | ML | MLb* | CHb* |
| -0.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | CH* | CH* | ML | ML | ML | MLb* | MLb* |
| -1.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | CH* | ML | ML | ML | MLb* | MLb* |
| -1.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | CH* | ML | ML | ML | MLb* |
| -1.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | CH* | CH* | ML | ML | MLb* |
| -1.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | CH | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | CH* | ML | ML | MLb* |
| -1.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | CH | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | CH* | CH* | ML | MLb* |
| -1.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | CH | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | ML | MLb* |
| -1.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | CH | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | CH* | ML |
| -1.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | ML |
| -1.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | CH* |
| -1.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* |
| -2.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* |
| -2.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* |
| -2.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -3.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CH | MLb | MLb | MLb | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |

Figure 9: Minimum LMSE among the $ML$ and $CH$ statistics.



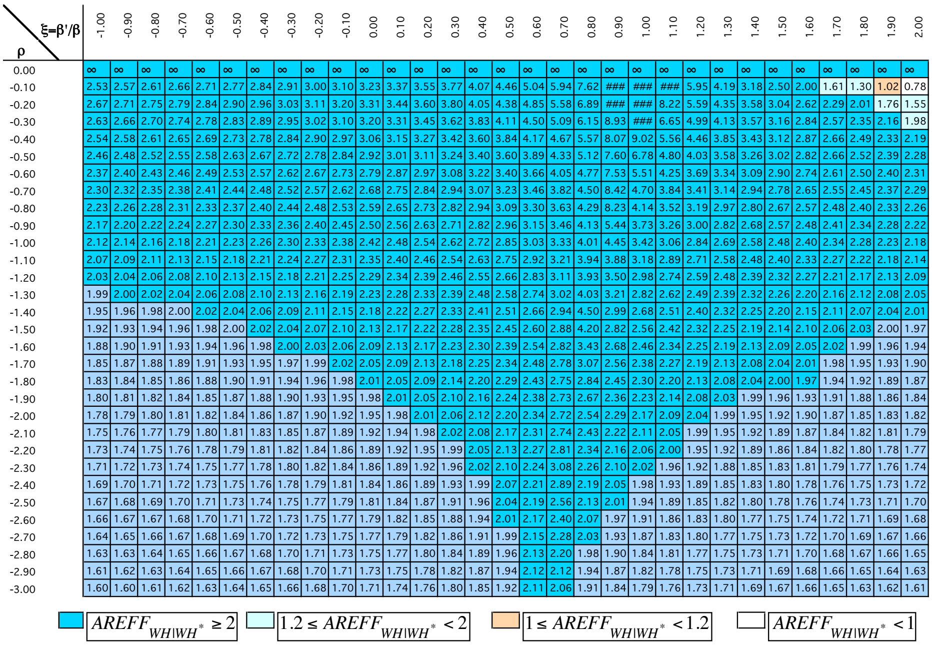
| ξ=β'/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -0.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | CHb* | CHb* | CHb* | CHb* | WHb* | WHb* |
| -0.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | CHb* | CHb* | CHb* | CHb* | CHb* |
| -0.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | CH* | ML | ML | ML | ML | MLb* | CHb* | CHb* | CHb* | CHb* |
| -0.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | MLb | CH | CH | ML | ML | ML | ML | ML | ML | ML | MLb* | CHb* | CHb* | CHb* |
| -0.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | CH | ML | ML | CH* | ML | ML | ML | ML | ML | CHb* | CHb* | CHb* |
| -0.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | WH | ML | ML | ML | ML | ML | ML | ML | ML | MLb* | CHb* | CHb* |
| -0.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | WH | ML | ML | ML | CH* | ML | ML | ML | ML | MLb* | MLb* | CHb* |
| -0.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | WH | ML | ML | ML | ML | CH* | ML | ML | ML | ML | MLb* | CHb* |
| -0.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | WH | ML | ML | ML | ML | CH* | CH* | ML | ML | ML | MLb* | MLb* |
| -1.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | MLb | MLb | CH | CH | WH | ML | ML | ML | ML | ML | CH* | ML | ML | ML | MLb* | MLb* |
| -1.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | CH* | ML | ML | ML | MLb* |
| -1.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | CH* | CH* | ML | ML | MLb* |
| -1.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | CH* | ML | ML | MLb* |
| -1.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | CH* | CH* | ML | MLb* |
| -1.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | CH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | ML | MLb* |
| -1.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | CH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | CH* | ML |
| -1.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | ML |
| -1.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* | CH* |
| -1.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* |
| -2.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* |
| -2.10 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | CH* |
| -2.20 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.30 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | MLb | CH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.40 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | WHb | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.50 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | WHb | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.60 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | WHb | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.70 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | WHb | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.80 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | CH | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -2.90 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | CH | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |
| -3.00 | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | CHb | WHb | CH | MLb | WH | WH | WH | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML | ML |

Figure 10: Minimum LMSE among the $ML$, $CH$ and $WH$ statistics.

still an open subject in the area of statistics of extremes, out of the scope of this paper.

# 5 Some additional comparisons

The $AREFF_{WH|WH^*}$ indicator is presented in Figure 11.



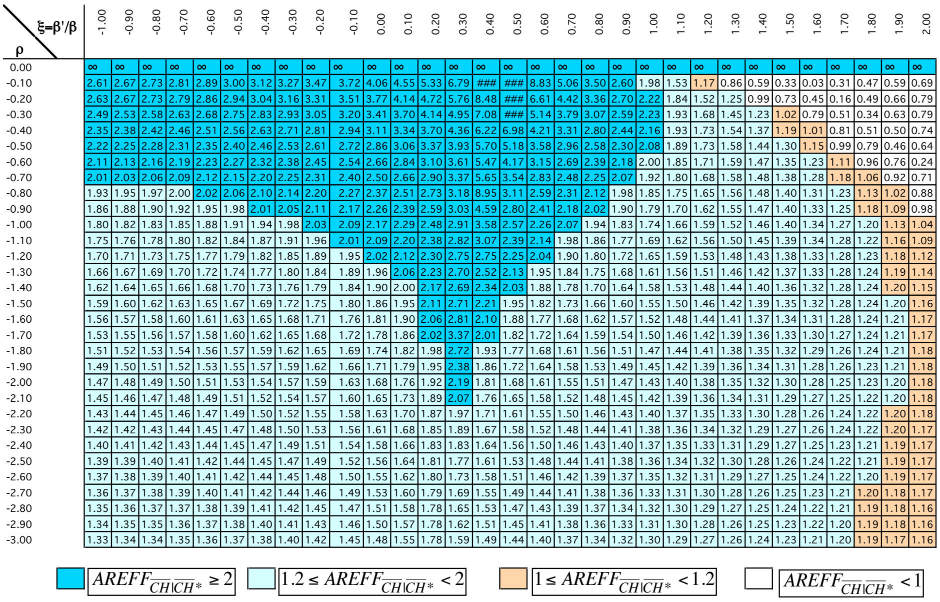
| ξ=β'/β<br>ρ | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 2.53 | 2.57 | 2.61 | 2.66 | 2.71 | 2.77 | 2.84 | 2.91 | 3.00 | 3.10 | 3.23 | 3.37 | 3.55 | 3.77 | 4.07 | 4.46 | 5.04 | 5.94 | 7.62 | ### | ### | ### | 5.95 | 4.19 | 3.18 | 2.50 | 2.00 | 1.61 | 1.30 | 1.02 | 0.78 |
| -0.20 | 2.67 | 2.71 | 2.75 | 2.79 | 2.84 | 2.90 | 2.96 | 3.03 | 3.11 | 3.20 | 3.31 | 3.44 | 3.60 | 3.80 | 4.05 | 4.38 | 4.85 | 5.58 | 6.89 | ### | ### | 8.22 | 5.59 | 4.35 | 3.58 | 3.04 | 2.62 | 2.29 | 2.01 | 1.76 | 1.55 |
| -0.30 | 2.63 | 2.66 | 2.70 | 2.74 | 2.78 | 2.83 | 2.89 | 2.95 | 3.02 | 3.10 | 3.20 | 3.31 | 3.45 | 3.62 | 3.83 | 4.11 | 4.50 | 5.09 | 6.15 | 8.93 | ### | 6.65 | 4.99 | 4.13 | 3.57 | 3.16 | 2.84 | 2.57 | 2.35 | 2.16 | 1.98 |
| -0.40 | 2.54 | 2.58 | 2.61 | 2.65 | 2.69 | 2.73 | 2.78 | 2.84 | 2.90 | 2.97 | 3.06 | 3.15 | 3.27 | 3.42 | 3.60 | 3.84 | 4.17 | 4.67 | 5.57 | 8.07 | 9.02 | 5.56 | 4.46 | 3.85 | 3.43 | 3.12 | 2.87 | 2.66 | 2.49 | 2.33 | 2.19 |
| -0.50 | 2.46 | 2.48 | 2.52 | 2.55 | 2.58 | 2.63 | 2.67 | 2.72 | 2.78 | 2.84 | 2.92 | 3.01 | 3.11 | 3.24 | 3.40 | 3.60 | 3.89 | 4.33 | 5.12 | 7.60 | 6.78 | 4.80 | 4.03 | 3.58 | 3.26 | 3.02 | 2.82 | 2.66 | 2.52 | 2.39 | 2.28 |
| -0.60 | 2.37 | 2.40 | 2.43 | 2.46 | 2.49 | 2.53 | 2.57 | 2.62 | 2.67 | 2.73 | 2.79 | 2.87 | 2.97 | 3.08 | 3.22 | 3.40 | 3.66 | 4.05 | 4.77 | 7.53 | 5.51 | 4.25 | 3.69 | 3.34 | 3.09 | 2.90 | 2.74 | 2.61 | 2.50 | 2.40 | 2.31 |
| -0.70 | 2.30 | 2.32 | 2.35 | 2.38 | 2.41 | 2.44 | 2.48 | 2.52 | 2.57 | 2.62 | 2.68 | 2.75 | 2.84 | 2.94 | 3.07 | 3.23 | 3.46 | 3.82 | 4.50 | 8.42 | 4.70 | 3.84 | 3.41 | 3.14 | 2.94 | 2.78 | 2.65 | 2.55 | 2.45 | 2.37 | 2.29 |
| -0.80 | 2.23 | 2.26 | 2.28 | 2.31 | 2.33 | 2.37 | 2.40 | 2.44 | 2.48 | 2.53 | 2.59 | 2.65 | 2.73 | 2.82 | 2.94 | 3.09 | 3.30 | 3.63 | 4.29 | 8.23 | 4.14 | 3.52 | 3.19 | 2.97 | 2.80 | 2.67 | 2.57 | 2.48 | 2.40 | 2.33 | 2.26 |
| -0.90 | 2.17 | 2.20 | 2.22 | 2.24 | 2.27 | 2.30 | 2.33 | 2.36 | 2.40 | 2.45 | 2.50 | 2.56 | 2.63 | 2.71 | 2.82 | 2.96 | 3.15 | 3.46 | 4.13 | 5.44 | 3.73 | 3.26 | 3.00 | 2.82 | 2.68 | 2.57 | 2.48 | 2.41 | 2.34 | 2.28 | 2.22 |
| -1.00 | 2.12 | 2.14 | 2.16 | 2.18 | 2.21 | 2.23 | 2.26 | 2.30 | 2.33 | 2.38 | 2.42 | 2.48 | 2.54 | 2.62 | 2.72 | 2.85 | 3.03 | 3.33 | 4.01 | 4.45 | 3.42 | 3.06 | 2.84 | 2.69 | 2.58 | 2.48 | 2.40 | 2.34 | 2.28 | 2.23 | 2.18 |
| -1.10 | 2.07 | 2.09 | 2.11 | 2.13 | 2.15 | 2.18 | 2.21 | 2.24 | 2.27 | 2.31 | 2.35 | 2.40 | 2.46 | 2.54 | 2.63 | 2.75 | 2.92 | 3.21 | 3.94 | 3.88 | 3.18 | 2.89 | 2.71 | 2.58 | 2.48 | 2.40 | 2.33 | 2.27 | 2.22 | 2.18 | 2.14 |
| -1.20 | 2.03 | 2.04 | 2.06 | 2.08 | 2.10 | 2.13 | 2.15 | 2.18 | 2.21 | 2.25 | 2.29 | 2.34 | 2.39 | 2.46 | 2.55 | 2.66 | 2.83 | 3.11 | 3.93 | 3.50 | 2.98 | 2.74 | 2.59 | 2.48 | 2.39 | 2.32 | 2.27 | 2.21 | 2.17 | 2.13 | 2.09 |
| -1.30 | 1.99 | 2.00 | 2.02 | 2.04 | 2.06 | 2.08 | 2.10 | 2.13 | 2.16 | 2.19 | 2.23 | 2.28 | 2.33 | 2.39 | 2.48 | 2.58 | 2.74 | 3.02 | 4.03 | 3.21 | 2.82 | 2.62 | 2.49 | 2.39 | 2.32 | 2.26 | 2.20 | 2.16 | 2.12 | 2.08 | 2.05 |
| -1.40 | 1.95 | 1.96 | 1.98 | 2.00 | 2.02 | 2.04 | 2.06 | 2.09 | 2.11 | 2.15 | 2.18 | 2.22 | 2.27 | 2.33 | 2.41 | 2.51 | 2.66 | 2.94 | 4.50 | 2.99 | 2.68 | 2.51 | 2.40 | 2.32 | 2.25 | 2.20 | 2.15 | 2.11 | 2.07 | 2.04 | 2.01 |
| -1.50 | 1.92 | 1.93 | 1.94 | 1.96 | 1.98 | 2.00 | 2.02 | 2.04 | 2.07 | 2.10 | 2.13 | 2.17 | 2.22 | 2.28 | 2.35 | 2.45 | 2.60 | 2.88 | 4.20 | 2.82 | 2.56 | 2.42 | 2.32 | 2.25 | 2.19 | 2.14 | 2.10 | 2.06 | 2.03 | 2.00 | 1.97 |
| -1.60 | 1.88 | 1.90 | 1.91 | 1.93 | 1.94 | 1.96 | 1.98 | 2.00 | 2.03 | 2.06 | 2.09 | 2.13 | 2.17 | 2.23 | 2.30 | 2.39 | 2.54 | 2.82 | 3.43 | 2.68 | 2.46 | 2.34 | 2.25 | 2.19 | 2.13 | 2.09 | 2.05 | 2.02 | 1.99 | 1.96 | 1.94 |
| -1.70 | 1.85 | 1.87 | 1.88 | 1.89 | 1.91 | 1.93 | 1.95 | 1.97 | 1.99 | 2.02 | 2.05 | 2.09 | 2.13 | 2.18 | 2.25 | 2.34 | 2.48 | 2.78 | 3.07 | 2.56 | 2.38 | 2.27 | 2.19 | 2.13 | 2.08 | 2.04 | 2.01 | 1.98 | 1.95 | 1.93 | 1.90 |
| -1.80 | 1.83 | 1.84 | 1.85 | 1.86 | 1.88 | 1.90 | 1.91 | 1.94 | 1.96 | 1.98 | 2.01 | 2.05 | 2.09 | 2.14 | 2.20 | 2.29 | 2.43 | 2.75 | 2.84 | 2.45 | 2.30 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 | 1.94 | 1.92 | 1.89 | 1.87 |
| -1.90 | 1.80 | 1.81 | 1.82 | 1.84 | 1.85 | 1.87 | 1.88 | 1.90 | 1.93 | 1.95 | 1.98 | 2.01 | 2.05 | 2.10 | 2.16 | 2.24 | 2.38 | 2.73 | 2.67 | 2.36 | 2.23 | 2.14 | 2.08 | 2.03 | 1.99 | 1.96 | 1.93 | 1.91 | 1.88 | 1.86 | 1.84 |
| -2.00 | 1.78 | 1.79 | 1.80 | 1.81 | 1.82 | 1.84 | 1.86 | 1.87 | 1.90 | 1.92 | 1.95 | 1.98 | 2.01 | 2.06 | 2.12 | 2.20 | 2.34 | 2.72 | 2.54 | 2.29 | 2.17 | 2.09 | 2.04 | 1.99 | 1.95 | 1.92 | 1.90 | 1.87 | 1.85 | 1.83 | 1.82 |
| -2.10 | 1.75 | 1.76 | 1.77 | 1.79 | 1.80 | 1.81 | 1.83 | 1.85 | 1.87 | 1.89 | 1.92 | 1.94 | 1.98 | 2.02 | 2.08 | 2.17 | 2.31 | 2.74 | 2.43 | 2.22 | 2.11 | 2.05 | 1.99 | 1.95 | 1.92 | 1.89 | 1.87 | 1.84 | 1.82 | 1.81 | 1.79 |
| -2.20 | 1.73 | 1.74 | 1.75 | 1.76 | 1.78 | 1.79 | 1.81 | 1.82 | 1.84 | 1.86 | 1.89 | 1.92 | 1.95 | 1.99 | 2.05 | 2.13 | 2.27 | 2.81 | 2.34 | 2.16 | 2.06 | 2.00 | 1.95 | 1.92 | 1.89 | 1.86 | 1.84 | 1.82 | 1.80 | 1.78 | 1.77 |
| -2.30 | 1.71 | 1.72 | 1.73 | 1.74 | 1.75 | 1.77 | 1.78 | 1.80 | 1.82 | 1.84 | 1.86 | 1.89 | 1.92 | 1.96 | 2.02 | 2.10 | 2.24 | 3.08 | 2.26 | 2.10 | 2.02 | 1.96 | 1.92 | 1.88 | 1.85 | 1.83 | 1.81 | 1.79 | 1.77 | 1.76 | 1.74 |
| -2.40 | 1.69 | 1.70 | 1.71 | 1.72 | 1.73 | 1.75 | 1.76 | 1.78 | 1.79 | 1.81 | 1.84 | 1.86 | 1.89 | 1.93 | 1.99 | 2.07 | 2.21 | 2.89 | 2.19 | 2.05 | 1.98 | 1.93 | 1.89 | 1.85 | 1.83 | 1.80 | 1.78 | 1.76 | 1.75 | 1.73 | 1.72 |
| -2.50 | 1.67 | 1.68 | 1.69 | 1.70 | 1.71 | 1.73 | 1.74 | 1.75 | 1.77 | 1.79 | 1.81 | 1.84 | 1.87 | 1.91 | 1.96 | 2.04 | 2.19 | 2.56 | 2.13 | 2.01 | 1.94 | 1.89 | 1.85 | 1.82 | 1.80 | 1.78 | 1.76 | 1.74 | 1.73 | 1.71 | 1.70 |
| -2.60 | 1.66 | 1.67 | 1.67 | 1.68 | 1.70 | 1.71 | 1.72 | 1.73 | 1.75 | 1.77 | 1.79 | 1.82 | 1.85 | 1.88 | 1.94 | 2.01 | 2.17 | 2.40 | 2.07 | 1.97 | 1.91 | 1.86 | 1.83 | 1.80 | 1.77 | 1.75 | 1.74 | 1.72 | 1.71 | 1.69 | 1.68 |
| -2.70 | 1.64 | 1.65 | 1.66 | 1.67 | 1.68 | 1.69 | 1.70 | 1.72 | 1.73 | 1.75 | 1.77 | 1.79 | 1.82 | 1.86 | 1.91 | 1.99 | 2.15 | 2.28 | 2.03 | 1.93 | 1.87 | 1.83 | 1.80 | 1.77 | 1.75 | 1.73 | 1.72 | 1.70 | 1.69 | 1.67 | 1.66 |
| -2.80 | 1.63 | 1.63 | 1.64 | 1.65 | 1.66 | 1.67 | 1.68 | 1.70 | 1.71 | 1.73 | 1.75 | 1.77 | 1.80 | 1.84 | 1.89 | 1.96 | 2.13 | 2.20 | 1.98 | 1.90 | 1.84 | 1.81 | 1.77 | 1.75 | 1.73 | 1.71 | 1.70 | 1.68 | 1.67 | 1.66 | 1.65 |
| -2.90 | 1.61 | 1.62 | 1.63 | 1.64 | 1.65 | 1.66 | 1.67 | 1.68 | 1.70 | 1.71 | 1.73 | 1.75 | 1.78 | 1.82 | 1.87 | 1.94 | 2.12 | 2.12 | 1.94 | 1.87 | 1.82 | 1.78 | 1.75 | 1.73 | 1.71 | 1.69 | 1.68 | 1.66 | 1.65 | 1.64 | 1.63 |
| -3.00 | 1.60 | 1.60 | 1.61 | 1.62 | 1.63 | 1.64 | 1.65 | 1.66 | 1.68 | 1.70 | 1.71 | 1.74 | 1.76 | 1.80 | 1.85 | 1.92 | 2.11 | 2.06 | 1.91 | 1.84 | 1.79 | 1.76 | 1.73 | 1.71 | 1.69 | 1.67 | 1.66 | 1.65 | 1.63 | 1.62 | 1.61 |

🟦 $AREFF_{WH|WH^*} \geq 2$ ▢ $1.2 \leq AREFF_{WH|WH^*} < 2$ 🟧 $1 \leq AREFF_{WH|WH^*} < 1.2$ □ $AREFF_{WH|WH^*} < 1$

Figure 11: $AREFF_{WH|WH^*}$, in the $(\xi, \rho)$-plane.

We next present the relative behaviour of $\overline{UH}$ and $\overline{UH}^\star$ for $UH = CH, ML$ and $WH$, in Figures 12, 13 and 14, respectively.

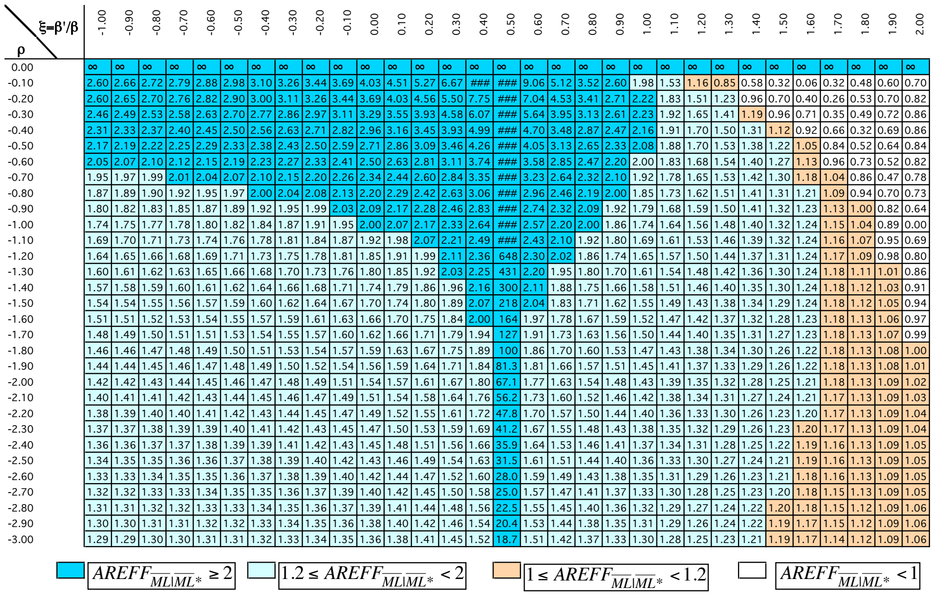
Finally, in Figures 15, 16 and 17, we provide the comparative behaviour of $\overline{UH}$ and $UH^*$, again for $UH = CH, ML$ and $WH$, respectively.



| ρ \ ξ=β'/β | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 2.61 | 2.67 | 2.73 | 2.81 | 2.89 | 3.00 | 3.12 | 3.27 | 3.47 | 3.72 | 4.06 | 4.55 | 5.33 | 6.79 | ### | ### | 8.83 | 5.06 | 3.50 | 2.60 | 1.98 | 1.53 | 1.17 | 0.86 | 0.59 | 0.33 | 0.03 | 0.31 | 0.47 | 0.59 | 0.69 |
| -0.20 | 2.63 | 2.67 | 2.73 | 2.79 | 2.86 | 2.94 | 3.04 | 3.16 | 3.31 | 3.51 | 3.77 | 4.14 | 4.72 | 5.76 | 8.48 | ### | 6.61 | 4.42 | 3.36 | 2.70 | 2.22 | 1.84 | 1.52 | 1.25 | 0.99 | 0.73 | 0.45 | 0.16 | 0.49 | 0.66 | 0.79 |
| -0.30 | 2.49 | 2.53 | 2.58 | 2.63 | 2.68 | 2.75 | 2.83 | 2.93 | 3.05 | 3.20 | 3.41 | 3.70 | 4.14 | 4.95 | 7.08 | ### | 5.14 | 3.79 | 3.07 | 2.59 | 2.23 | 1.93 | 1.68 | 1.45 | 1.23 | 1.02 | 0.79 | 0.51 | 0.34 | 0.63 | 0.79 |
| -0.40 | 2.35 | 2.38 | 2.42 | 2.46 | 2.51 | 2.56 | 2.63 | 2.71 | 2.81 | 2.94 | 3.11 | 3.34 | 3.70 | 4.36 | 6.22 | 6.98 | 4.21 | 3.31 | 2.80 | 2.44 | 2.16 | 1.93 | 1.73 | 1.54 | 1.37 | 1.19 | 1.01 | 0.81 | 0.51 | 0.50 | 0.74 |
| -0.50 | 2.22 | 2.25 | 2.28 | 2.31 | 2.35 | 2.40 | 2.46 | 2.53 | 2.61 | 2.72 | 2.86 | 3.06 | 3.37 | 3.93 | 5.70 | 5.18 | 3.58 | 2.96 | 2.58 | 2.30 | 2.08 | 1.89 | 1.73 | 1.58 | 1.44 | 1.30 | 1.15 | 0.99 | 0.79 | 0.46 | 0.64 |
| -0.60 | 2.11 | 2.13 | 2.16 | 2.19 | 2.23 | 2.27 | 2.32 | 2.38 | 2.45 | 2.54 | 2.66 | 2.84 | 3.10 | 3.61 | 5.47 | 4.17 | 3.15 | 2.69 | 2.39 | 2.18 | 2.00 | 1.85 | 1.71 | 1.59 | 1.47 | 1.35 | 1.23 | 1.11 | 0.96 | 0.76 | 0.24 |
| -0.70 | 2.01 | 2.03 | 2.06 | 2.09 | 2.12 | 2.15 | 2.20 | 2.25 | 2.31 | 2.40 | 2.50 | 2.66 | 2.90 | 3.37 | 5.65 | 3.54 | 2.83 | 2.48 | 2.25 | 2.07 | 1.92 | 1.80 | 1.68 | 1.58 | 1.48 | 1.38 | 1.28 | 1.18 | 1.06 | 0.92 | 0.71 |
| -0.80 | 1.93 | 1.95 | 1.97 | 2.00 | 2.02 | 2.06 | 2.10 | 2.14 | 2.20 | 2.27 | 2.37 | 2.51 | 2.73 | 3.18 | 8.95 | 3.11 | 2.59 | 2.31 | 2.12 | 1.98 | 1.85 | 1.75 | 1.65 | 1.56 | 1.48 | 1.40 | 1.31 | 1.23 | 1.13 | 1.02 | 0.88 |
| -0.90 | 1.86 | 1.88 | 1.90 | 1.92 | 1.95 | 1.98 | 2.01 | 2.05 | 2.11 | 2.17 | 2.26 | 2.39 | 2.59 | 3.03 | 4.59 | 2.80 | 2.41 | 2.18 | 2.02 | 1.90 | 1.79 | 1.70 | 1.62 | 1.55 | 1.47 | 1.40 | 1.33 | 1.25 | 1.18 | 1.09 | 0.98 |
| -1.00 | 1.80 | 1.82 | 1.83 | 1.85 | 1.88 | 1.91 | 1.94 | 1.98 | 2.03 | 2.09 | 2.17 | 2.29 | 2.48 | 2.91 | 3.58 | 2.57 | 2.26 | 2.07 | 1.94 | 1.83 | 1.74 | 1.66 | 1.59 | 1.52 | 1.46 | 1.40 | 1.34 | 1.27 | 1.20 | 1.13 | 1.04 |
| -1.10 | 1.75 | 1.76 | 1.78 | 1.80 | 1.82 | 1.84 | 1.87 | 1.91 | 1.96 | 2.01 | 2.09 | 2.20 | 2.38 | 2.82 | 3.07 | 2.39 | 2.14 | 1.98 | 1.86 | 1.77 | 1.69 | 1.62 | 1.56 | 1.50 | 1.45 | 1.39 | 1.34 | 1.28 | 1.22 | 1.16 | 1.09 |
| -1.20 | 1.70 | 1.71 | 1.73 | 1.75 | 1.77 | 1.79 | 1.82 | 1.85 | 1.89 | 1.95 | 2.02 | 2.12 | 2.30 | 2.75 | 2.75 | 2.25 | 2.04 | 1.90 | 1.80 | 1.72 | 1.65 | 1.59 | 1.53 | 1.48 | 1.43 | 1.38 | 1.33 | 1.28 | 1.23 | 1.18 | 1.12 |
| -1.30 | 1.66 | 1.67 | 1.69 | 1.70 | 1.72 | 1.74 | 1.77 | 1.80 | 1.84 | 1.89 | 1.96 | 2.06 | 2.23 | 2.70 | 2.52 | 2.13 | 1.95 | 1.84 | 1.75 | 1.68 | 1.61 | 1.56 | 1.51 | 1.46 | 1.42 | 1.37 | 1.33 | 1.28 | 1.24 | 1.19 | 1.14 |
| -1.40 | 1.62 | 1.64 | 1.65 | 1.66 | 1.68 | 1.70 | 1.73 | 1.76 | 1.79 | 1.84 | 1.90 | 2.00 | 2.17 | 2.69 | 2.34 | 2.03 | 1.88 | 1.78 | 1.70 | 1.64 | 1.58 | 1.53 | 1.48 | 1.44 | 1.40 | 1.36 | 1.32 | 1.28 | 1.24 | 1.20 | 1.15 |
| -1.50 | 1.59 | 1.60 | 1.62 | 1.63 | 1.65 | 1.67 | 1.69 | 1.72 | 1.75 | 1.80 | 1.86 | 1.95 | 2.11 | 2.71 | 2.21 | 1.95 | 1.82 | 1.73 | 1.66 | 1.60 | 1.55 | 1.50 | 1.46 | 1.42 | 1.39 | 1.35 | 1.32 | 1.28 | 1.24 | 1.20 | 1.16 |
| -1.60 | 1.56 | 1.57 | 1.58 | 1.60 | 1.61 | 1.63 | 1.65 | 1.68 | 1.71 | 1.76 | 1.81 | 1.90 | 2.06 | 2.81 | 2.10 | 1.88 | 1.77 | 1.68 | 1.62 | 1.57 | 1.52 | 1.48 | 1.44 | 1.41 | 1.37 | 1.34 | 1.31 | 1.28 | 1.24 | 1.21 | 1.17 |
| -1.70 | 1.53 | 1.55 | 1.56 | 1.57 | 1.58 | 1.60 | 1.62 | 1.65 | 1.68 | 1.72 | 1.78 | 1.86 | 2.02 | 3.37 | 2.01 | 1.82 | 1.72 | 1.64 | 1.59 | 1.54 | 1.50 | 1.46 | 1.42 | 1.39 | 1.36 | 1.33 | 1.30 | 1.27 | 1.24 | 1.21 | 1.17 |
| -1.80 | 1.51 | 1.52 | 1.53 | 1.54 | 1.56 | 1.57 | 1.59 | 1.62 | 1.65 | 1.69 | 1.74 | 1.82 | 1.98 | 2.72 | 1.93 | 1.77 | 1.68 | 1.61 | 1.56 | 1.51 | 1.47 | 1.44 | 1.41 | 1.38 | 1.35 | 1.32 | 1.29 | 1.26 | 1.24 | 1.21 | 1.18 |
| -1.90 | 1.49 | 1.50 | 1.51 | 1.52 | 1.53 | 1.55 | 1.57 | 1.59 | 1.62 | 1.66 | 1.71 | 1.79 | 1.95 | 2.38 | 1.86 | 1.72 | 1.64 | 1.58 | 1.53 | 1.49 | 1.45 | 1.42 | 1.39 | 1.36 | 1.34 | 1.31 | 1.28 | 1.26 | 1.23 | 1.21 | 1.18 |
| -2.00 | 1.47 | 1.48 | 1.49 | 1.50 | 1.51 | 1.53 | 1.54 | 1.57 | 1.59 | 1.63 | 1.68 | 1.76 | 1.92 | 2.19 | 1.81 | 1.68 | 1.61 | 1.55 | 1.51 | 1.47 | 1.43 | 1.40 | 1.38 | 1.35 | 1.32 | 1.30 | 1.28 | 1.25 | 1.23 | 1.20 | 1.18 |
| -2.10 | 1.45 | 1.46 | 1.47 | 1.48 | 1.49 | 1.51 | 1.52 | 1.54 | 1.57 | 1.60 | 1.65 | 1.73 | 1.89 | 2.07 | 1.76 | 1.65 | 1.58 | 1.52 | 1.48 | 1.45 | 1.42 | 1.39 | 1.36 | 1.34 | 1.31 | 1.29 | 1.27 | 1.25 | 1.22 | 1.20 | 1.18 |
| -2.20 | 1.43 | 1.44 | 1.45 | 1.46 | 1.47 | 1.49 | 1.50 | 1.52 | 1.55 | 1.58 | 1.63 | 1.70 | 1.87 | 1.97 | 1.71 | 1.61 | 1.55 | 1.50 | 1.46 | 1.43 | 1.40 | 1.37 | 1.35 | 1.33 | 1.30 | 1.28 | 1.26 | 1.24 | 1.22 | 1.20 | 1.18 |
| -2.30 | 1.42 | 1.42 | 1.43 | 1.44 | 1.45 | 1.47 | 1.48 | 1.50 | 1.53 | 1.56 | 1.61 | 1.68 | 1.85 | 1.89 | 1.67 | 1.58 | 1.52 | 1.48 | 1.44 | 1.41 | 1.38 | 1.36 | 1.34 | 1.32 | 1.29 | 1.27 | 1.26 | 1.24 | 1.22 | 1.20 | 1.17 |
| -2.40 | 1.40 | 1.41 | 1.42 | 1.43 | 1.44 | 1.45 | 1.47 | 1.49 | 1.51 | 1.54 | 1.58 | 1.66 | 1.83 | 1.83 | 1.64 | 1.56 | 1.50 | 1.46 | 1.43 | 1.40 | 1.37 | 1.35 | 1.33 | 1.31 | 1.29 | 1.27 | 1.25 | 1.23 | 1.21 | 1.19 | 1.17 |
| -2.50 | 1.39 | 1.39 | 1.40 | 1.41 | 1.42 | 1.44 | 1.45 | 1.47 | 1.49 | 1.52 | 1.56 | 1.64 | 1.81 | 1.77 | 1.61 | 1.53 | 1.48 | 1.44 | 1.41 | 1.38 | 1.36 | 1.34 | 1.32 | 1.30 | 1.28 | 1.26 | 1.24 | 1.22 | 1.21 | 1.19 | 1.17 |
| -2.60 | 1.37 | 1.38 | 1.39 | 1.40 | 1.41 | 1.42 | 1.44 | 1.45 | 1.48 | 1.50 | 1.55 | 1.62 | 1.80 | 1.73 | 1.58 | 1.51 | 1.46 | 1.42 | 1.39 | 1.37 | 1.35 | 1.32 | 1.31 | 1.29 | 1.27 | 1.25 | 1.24 | 1.22 | 1.20 | 1.19 | 1.17 |
| -2.70 | 1.36 | 1.37 | 1.38 | 1.39 | 1.40 | 1.41 | 1.42 | 1.44 | 1.46 | 1.49 | 1.53 | 1.60 | 1.79 | 1.69 | 1.55 | 1.49 | 1.44 | 1.41 | 1.38 | 1.36 | 1.33 | 1.31 | 1.30 | 1.28 | 1.26 | 1.25 | 1.23 | 1.21 | 1.20 | 1.18 | 1.17 |
| -2.80 | 1.35 | 1.36 | 1.37 | 1.37 | 1.38 | 1.39 | 1.41 | 1.42 | 1.45 | 1.47 | 1.51 | 1.58 | 1.78 | 1.65 | 1.53 | 1.47 | 1.43 | 1.39 | 1.37 | 1.34 | 1.32 | 1.30 | 1.29 | 1.27 | 1.25 | 1.24 | 1.22 | 1.21 | 1.19 | 1.18 | 1.16 |
| -2.90 | 1.34 | 1.35 | 1.35 | 1.36 | 1.37 | 1.38 | 1.40 | 1.41 | 1.43 | 1.46 | 1.50 | 1.57 | 1.78 | 1.62 | 1.51 | 1.45 | 1.41 | 1.38 | 1.36 | 1.33 | 1.31 | 1.30 | 1.28 | 1.26 | 1.25 | 1.23 | 1.22 | 1.20 | 1.19 | 1.18 | 1.16 |
| -3.00 | 1.33 | 1.34 | 1.34 | 1.35 | 1.36 | 1.37 | 1.38 | 1.40 | 1.42 | 1.45 | 1.48 | 1.55 | 1.78 | 1.59 | 1.49 | 1.44 | 1.40 | 1.37 | 1.34 | 1.32 | 1.30 | 1.29 | 1.27 | 1.26 | 1.24 | 1.23 | 1.21 | 1.20 | 1.19 | 1.17 | 1.16 |

🟦 $AREFF_{\overline{CH}|\overline{CH}^*} \geq 2$ ░ $1.2 \leq AREFF_{\overline{CH}|\overline{CH}^*} < 2$ 🟧 $1 \leq AREFF_{\overline{CH}|\overline{CH}^*} < 1.2$ ☐ $AREFF_{\overline{CH}|\overline{CH}^*} < 1$

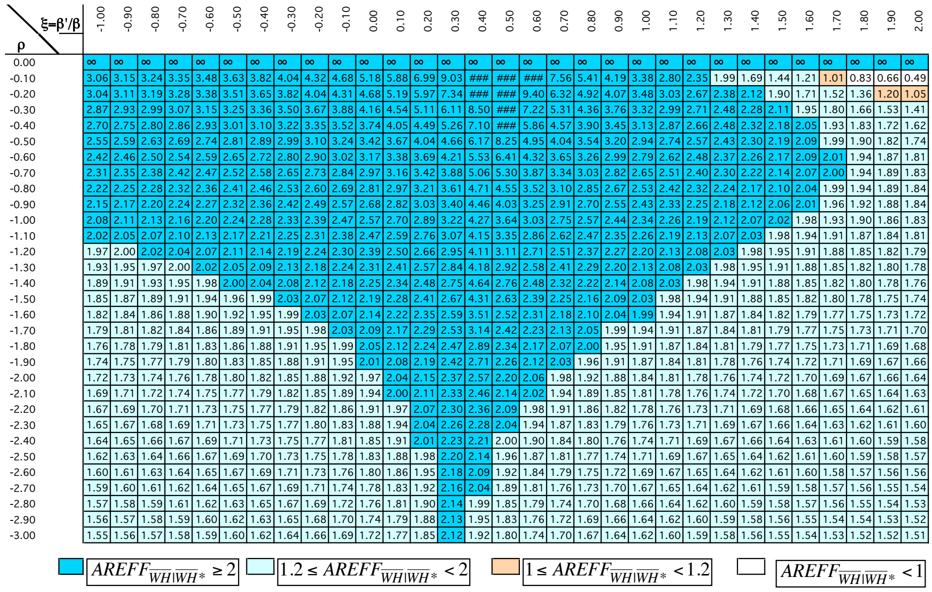
Figure 12: $AREFF_{\overline{CH}|\overline{CH}^*}$, in the $(\xi, \rho)$-plane.



| ρ \ ξ=β'/β | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 2.60 | 2.66 | 2.72 | 2.79 | 2.88 | 2.98 | 3.10 | 3.26 | 3.44 | 3.69 | 4.03 | 4.51 | 5.27 | 6.67 | ### | ### | 9.06 | 5.12 | 3.52 | 2.60 | 1.98 | 1.53 | 1.16 | 0.85 | 0.58 | 0.32 | 0.06 | 0.32 | 0.48 | 0.60 | 0.70 |
| -0.20 | 2.60 | 2.65 | 2.70 | 2.76 | 2.82 | 2.90 | 3.00 | 3.11 | 3.26 | 3.44 | 3.69 | 4.03 | 4.56 | 5.50 | 7.75 | ### | 7.04 | 4.53 | 3.41 | 2.71 | 2.22 | 1.83 | 1.51 | 1.23 | 0.96 | 0.70 | 0.40 | 0.26 | 0.53 | 0.70 | 0.82 |
| -0.30 | 2.46 | 2.49 | 2.53 | 2.58 | 2.63 | 2.70 | 2.77 | 2.86 | 2.97 | 3.11 | 3.29 | 3.55 | 3.93 | 4.58 | 6.07 | ### | 5.64 | 3.95 | 3.13 | 2.61 | 2.23 | 1.92 | 1.65 | 1.41 | 1.19 | 0.96 | 0.71 | 0.35 | 0.49 | 0.72 | 0.86 |
| -0.40 | 2.31 | 2.33 | 2.37 | 2.40 | 2.45 | 2.50 | 2.56 | 2.63 | 2.71 | 2.82 | 2.96 | 3.16 | 3.45 | 3.93 | 4.99 | ### | 4.70 | 3.48 | 2.87 | 2.47 | 2.16 | 1.91 | 1.70 | 1.50 | 1.31 | 1.12 | 0.92 | 0.66 | 0.32 | 0.69 | 0.86 |
| -0.50 | 2.17 | 2.19 | 2.22 | 2.25 | 2.29 | 2.33 | 2.38 | 2.43 | 2.50 | 2.59 | 2.71 | 2.86 | 3.09 | 3.46 | 4.26 | ### | 4.05 | 3.13 | 2.65 | 2.33 | 2.08 | 1.88 | 1.70 | 1.53 | 1.38 | 1.22 | 1.05 | 0.84 | 0.52 | 0.64 | 0.84 |
| -0.60 | 2.05 | 2.07 | 2.10 | 2.12 | 2.15 | 2.19 | 2.23 | 2.27 | 2.33 | 2.41 | 2.50 | 2.63 | 2.81 | 3.11 | 3.74 | ### | 3.58 | 2.85 | 2.47 | 2.20 | 2.00 | 1.83 | 1.68 | 1.54 | 1.40 | 1.27 | 1.13 | 0.96 | 0.73 | 0.52 | 0.82 |
| -0.70 | 1.95 | 1.97 | 1.99 | 2.01 | 2.04 | 2.07 | 2.10 | 2.15 | 2.20 | 2.26 | 2.34 | 2.44 | 2.60 | 2.84 | 3.35 | ### | 3.23 | 2.64 | 2.32 | 2.10 | 1.92 | 1.78 | 1.65 | 1.53 | 1.42 | 1.30 | 1.18 | 1.04 | 0.86 | 0.47 | 0.78 |
| -0.80 | 1.87 | 1.89 | 1.90 | 1.92 | 1.95 | 1.97 | 2.00 | 2.04 | 2.08 | 2.13 | 2.20 | 2.29 | 2.42 | 2.63 | 3.06 | ### | 2.96 | 2.46 | 2.19 | 2.00 | 1.85 | 1.73 | 1.62 | 1.51 | 1.41 | 1.31 | 1.21 | 1.09 | 0.94 | 0.70 | 0.73 |
| -0.90 | 1.80 | 1.82 | 1.83 | 1.85 | 1.87 | 1.89 | 1.92 | 1.95 | 1.99 | 2.03 | 2.09 | 2.17 | 2.28 | 2.46 | 2.83 | ### | 2.74 | 2.32 | 2.09 | 1.92 | 1.79 | 1.68 | 1.59 | 1.50 | 1.41 | 1.32 | 1.23 | 1.13 | 1.00 | 0.82 | 0.64 |
| -1.00 | 1.74 | 1.75 | 1.77 | 1.78 | 1.80 | 1.82 | 1.84 | 1.87 | 1.91 | 1.95 | 2.00 | 2.07 | 2.17 | 2.33 | 2.64 | ### | 2.57 | 2.20 | 2.00 | 1.86 | 1.74 | 1.64 | 1.56 | 1.48 | 1.40 | 1.32 | 1.24 | 1.15 | 1.04 | 0.89 | 0.00 |
| -1.10 | 1.69 | 1.70 | 1.71 | 1.73 | 1.74 | 1.76 | 1.78 | 1.81 | 1.84 | 1.87 | 1.92 | 1.98 | 2.07 | 2.21 | 2.49 | ### | 2.43 | 2.10 | 1.92 | 1.80 | 1.69 | 1.61 | 1.53 | 1.46 | 1.39 | 1.32 | 1.24 | 1.16 | 1.07 | 0.95 | 0.69 |
| -1.20 | 1.64 | 1.65 | 1.66 | 1.68 | 1.69 | 1.71 | 1.73 | 1.75 | 1.78 | 1.81 | 1.85 | 1.91 | 1.99 | 2.11 | 2.36 | 648 | 2.30 | 2.02 | 1.86 | 1.74 | 1.65 | 1.57 | 1.50 | 1.44 | 1.37 | 1.31 | 1.24 | 1.17 | 1.09 | 0.98 | 0.80 |
| -1.30 | 1.60 | 1.61 | 1.62 | 1.63 | 1.65 | 1.66 | 1.68 | 1.70 | 1.73 | 1.76 | 1.80 | 1.85 | 1.92 | 2.03 | 2.25 | 431 | 2.20 | 1.95 | 1.80 | 1.70 | 1.61 | 1.54 | 1.48 | 1.42 | 1.36 | 1.30 | 1.24 | 1.18 | 1.11 | 1.01 | 0.86 |
| -1.40 | 1.57 | 1.58 | 1.59 | 1.60 | 1.61 | 1.62 | 1.64 | 1.66 | 1.68 | 1.71 | 1.74 | 1.79 | 1.86 | 1.96 | 2.16 | 300 | 2.11 | 1.88 | 1.75 | 1.66 | 1.58 | 1.51 | 1.46 | 1.40 | 1.35 | 1.30 | 1.24 | 1.18 | 1.12 | 1.03 | 0.91 |
| -1.50 | 1.54 | 1.54 | 1.55 | 1.56 | 1.57 | 1.59 | 1.60 | 1.62 | 1.64 | 1.67 | 1.70 | 1.74 | 1.80 | 1.89 | 2.07 | 218 | 2.04 | 1.83 | 1.71 | 1.62 | 1.55 | 1.49 | 1.43 | 1.38 | 1.34 | 1.29 | 1.24 | 1.18 | 1.12 | 1.05 | 0.94 |
| -1.60 | 1.51 | 1.51 | 1.52 | 1.53 | 1.54 | 1.55 | 1.57 | 1.59 | 1.61 | 1.63 | 1.66 | 1.70 | 1.75 | 1.84 | 2.00 | 164 | 1.97 | 1.78 | 1.67 | 1.59 | 1.52 | 1.47 | 1.42 | 1.37 | 1.32 | 1.28 | 1.23 | 1.18 | 1.13 | 1.06 | 0.97 |
| -1.70 | 1.48 | 1.49 | 1.50 | 1.51 | 1.51 | 1.53 | 1.54 | 1.55 | 1.57 | 1.60 | 1.62 | 1.66 | 1.71 | 1.79 | 1.94 | 127 | 1.91 | 1.73 | 1.63 | 1.56 | 1.50 | 1.44 | 1.40 | 1.35 | 1.31 | 1.27 | 1.23 | 1.18 | 1.13 | 1.07 | 0.99 |
| -1.80 | 1.46 | 1.46 | 1.47 | 1.48 | 1.49 | 1.50 | 1.51 | 1.53 | 1.54 | 1.57 | 1.59 | 1.63 | 1.67 | 1.75 | 1.89 | 100 | 1.86 | 1.70 | 1.60 | 1.53 | 1.47 | 1.43 | 1.38 | 1.34 | 1.30 | 1.26 | 1.22 | 1.18 | 1.13 | 1.08 | 1.00 |
| -1.90 | 1.44 | 1.44 | 1.45 | 1.46 | 1.47 | 1.48 | 1.49 | 1.50 | 1.52 | 1.54 | 1.56 | 1.59 | 1.64 | 1.71 | 1.84 | 81.3 | 1.81 | 1.66 | 1.57 | 1.51 | 1.45 | 1.41 | 1.37 | 1.33 | 1.29 | 1.26 | 1.22 | 1.18 | 1.13 | 1.08 | 1.01 |
| -2.00 | 1.42 | 1.42 | 1.43 | 1.44 | 1.45 | 1.46 | 1.47 | 1.48 | 1.49 | 1.51 | 1.54 | 1.57 | 1.61 | 1.67 | 1.80 | 67.1 | 1.77 | 1.63 | 1.54 | 1.48 | 1.43 | 1.39 | 1.35 | 1.32 | 1.28 | 1.25 | 1.21 | 1.18 | 1.13 | 1.09 | 1.02 |
| -2.10 | 1.40 | 1.41 | 1.41 | 1.42 | 1.43 | 1.44 | 1.45 | 1.46 | 1.47 | 1.49 | 1.51 | 1.54 | 1.58 | 1.64 | 1.76 | 56.2 | 1.73 | 1.60 | 1.52 | 1.46 | 1.42 | 1.38 | 1.34 | 1.31 | 1.27 | 1.24 | 1.21 | 1.17 | 1.13 | 1.09 | 1.03 |
| -2.20 | 1.38 | 1.39 | 1.40 | 1.40 | 1.41 | 1.42 | 1.43 | 1.44 | 1.45 | 1.47 | 1.49 | 1.52 | 1.55 | 1.61 | 1.72 | 47.8 | 1.70 | 1.57 | 1.50 | 1.44 | 1.40 | 1.36 | 1.33 | 1.30 | 1.26 | 1.23 | 1.20 | 1.17 | 1.13 | 1.09 | 1.04 |
| -2.30 | 1.37 | 1.37 | 1.38 | 1.39 | 1.39 | 1.40 | 1.41 | 1.42 | 1.43 | 1.45 | 1.47 | 1.50 | 1.53 | 1.59 | 1.69 | 41.2 | 1.67 | 1.55 | 1.48 | 1.43 | 1.38 | 1.35 | 1.32 | 1.29 | 1.26 | 1.23 | 1.20 | 1.17 | 1.13 | 1.09 | 1.04 |
| -2.40 | 1.36 | 1.36 | 1.37 | 1.37 | 1.38 | 1.39 | 1.39 | 1.41 | 1.42 | 1.43 | 1.45 | 1.48 | 1.51 | 1.56 | 1.66 | 35.9 | 1.64 | 1.53 | 1.46 | 1.41 | 1.37 | 1.34 | 1.31 | 1.28 | 1.25 | 1.22 | 1.19 | 1.16 | 1.13 | 1.09 | 1.05 |
| -2.50 | 1.34 | 1.35 | 1.35 | 1.36 | 1.36 | 1.37 | 1.38 | 1.39 | 1.40 | 1.42 | 1.43 | 1.46 | 1.49 | 1.54 | 1.63 | 31.5 | 1.61 | 1.51 | 1.44 | 1.40 | 1.36 | 1.33 | 1.30 | 1.27 | 1.24 | 1.21 | 1.19 | 1.16 | 1.13 | 1.09 | 1.05 |
| -2.60 | 1.33 | 1.33 | 1.34 | 1.35 | 1.35 | 1.36 | 1.37 | 1.38 | 1.39 | 1.40 | 1.42 | 1.44 | 1.47 | 1.52 | 1.60 | 28.0 | 1.59 | 1.49 | 1.43 | 1.38 | 1.35 | 1.31 | 1.29 | 1.26 | 1.23 | 1.21 | 1.18 | 1.16 | 1.13 | 1.09 | 1.05 |
| -2.70 | 1.32 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.35 | 1.36 | 1.37 | 1.39 | 1.40 | 1.42 | 1.45 | 1.50 | 1.58 | 25.0 | 1.57 | 1.47 | 1.41 | 1.37 | 1.33 | 1.30 | 1.28 | 1.25 | 1.23 | 1.20 | 1.18 | 1.15 | 1.13 | 1.09 | 1.05 |
| -2.80 | 1.31 | 1.31 | 1.32 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.36 | 1.37 | 1.39 | 1.41 | 1.44 | 1.48 | 1.56 | 22.5 | 1.55 | 1.45 | 1.40 | 1.36 | 1.32 | 1.29 | 1.27 | 1.24 | 1.22 | 1.20 | 1.18 | 1.15 | 1.12 | 1.09 | 1.06 |
| -2.90 | 1.30 | 1.30 | 1.31 | 1.31 | 1.32 | 1.32 | 1.33 | 1.34 | 1.35 | 1.36 | 1.38 | 1.40 | 1.42 | 1.46 | 1.54 | 20.4 | 1.53 | 1.44 | 1.38 | 1.35 | 1.31 | 1.29 | 1.26 | 1.24 | 1.22 | 1.19 | 1.17 | 1.15 | 1.12 | 1.09 | 1.06 |
| -3.00 | 1.29 | 1.29 | 1.30 | 1.30 | 1.31 | 1.31 | 1.32 | 1.33 | 1.34 | 1.35 | 1.36 | 1.38 | 1.41 | 1.45 | 1.52 | 18.7 | 1.51 | 1.42 | 1.37 | 1.33 | 1.30 | 1.28 | 1.25 | 1.23 | 1.21 | 1.19 | 1.17 | 1.14 | 1.12 | 1.09 | 1.06 |

🟦 $AREFF_{\overline{ML}|\overline{ML}^*} \geq 2$ ░ $1.2 \leq AREFF_{\overline{ML}|\overline{ML}^*} < 2$ 🟧 $1 \leq AREFF_{\overline{ML}|\overline{ML}^*} < 1.2$ ☐ $AREFF_{\overline{ML}|\overline{ML}^*} < 1$

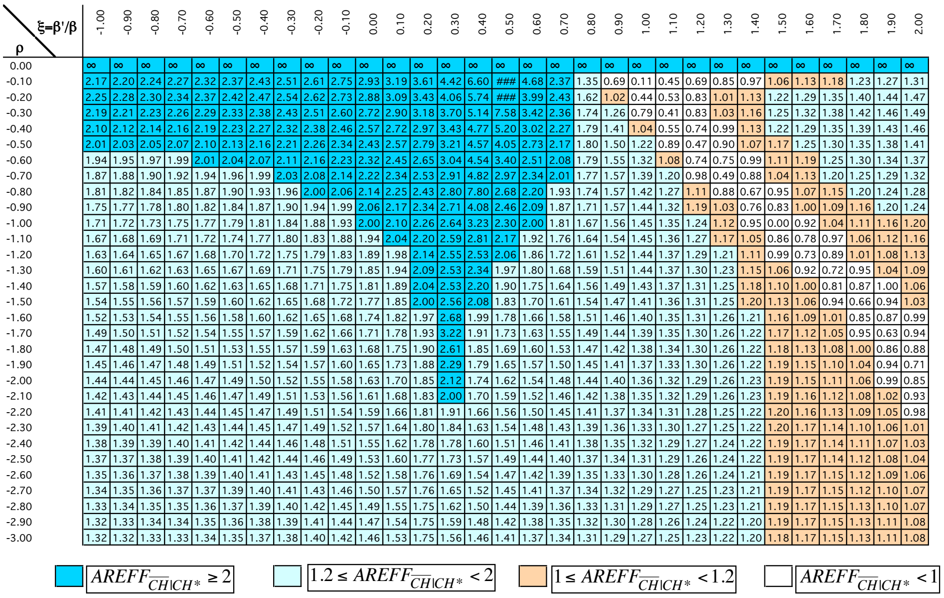
Figure 13: $AREFF_{\overline{ML}|\overline{ML}^*}$, in the $(\xi, \rho)$-plane.



| ρ \ ξ=β'/β | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 3.06 | 3.15 | 3.24 | 3.35 | 3.48 | 3.63 | 3.82 | 4.04 | 4.32 | 4.68 | 5.18 | 5.88 | 6.99 | 9.03 | ### | ### | ### | 7.56 | 5.41 | 4.19 | 3.38 | 2.80 | 2.35 | 1.99 | 1.69 | 1.44 | 1.21 | 1.01 | 0.83 | 0.66 | 0.49 |
| -0.20 | 3.04 | 3.11 | 3.19 | 3.28 | 3.38 | 3.51 | 3.65 | 3.82 | 4.04 | 4.31 | 4.68 | 5.19 | 5.97 | 7.34 | ### | ### | 9.40 | 6.32 | 4.92 | 4.07 | 3.48 | 3.03 | 2.67 | 2.38 | 2.12 | 1.90 | 1.71 | 1.52 | 1.36 | 1.20 | 1.05 |
| -0.30 | 2.87 | 2.93 | 2.99 | 3.07 | 3.15 | 3.25 | 3.36 | 3.50 | 3.67 | 3.88 | 4.16 | 4.54 | 5.11 | 6.11 | 8.50 | ### | 7.22 | 5.31 | 4.36 | 3.76 | 3.32 | 2.99 | 2.71 | 2.48 | 2.28 | 2.11 | 1.95 | 1.80 | 1.66 | 1.53 | 1.41 |
| -0.40 | 2.70 | 2.75 | 2.80 | 2.86 | 2.93 | 3.01 | 3.10 | 3.22 | 3.35 | 3.52 | 3.74 | 4.05 | 4.49 | 5.26 | 7.10 | ### | 5.86 | 4.57 | 3.90 | 3.45 | 3.13 | 2.87 | 2.66 | 2.48 | 2.32 | 2.18 | 2.05 | 1.93 | 1.83 | 1.72 | 1.62 |
| -0.50 | 2.55 | 2.59 | 2.63 | 2.69 | 2.74 | 2.81 | 2.89 | 2.99 | 3.10 | 3.24 | 3.42 | 3.67 | 4.04 | 4.66 | 6.17 | 8.25 | 4.95 | 4.04 | 3.54 | 3.20 | 2.94 | 2.74 | 2.57 | 2.43 | 2.30 | 2.19 | 2.09 | 1.99 | 1.90 | 1.82 | 1.74 |
| -0.60 | 2.42 | 2.46 | 2.50 | 2.54 | 2.59 | 2.65 | 2.72 | 2.80 | 2.90 | 3.02 | 3.17 | 3.38 | 3.69 | 4.21 | 5.53 | 6.41 | 4.32 | 3.65 | 3.26 | 2.99 | 2.79 | 2.62 | 2.48 | 2.37 | 2.26 | 2.17 | 2.09 | 2.01 | 1.94 | 1.87 | 1.81 |
| -0.70 | 2.31 | 2.35 | 2.38 | 2.42 | 2.47 | 2.52 | 2.58 | 2.65 | 2.73 | 2.84 | 2.97 | 3.16 | 3.42 | 3.88 | 5.06 | 5.30 | 3.87 | 3.34 | 3.03 | 2.82 | 2.65 | 2.51 | 2.40 | 2.30 | 2.22 | 2.14 | 2.07 | 2.00 | 1.94 | 1.89 | 1.83 |
| -0.80 | 2.22 | 2.25 | 2.28 | 2.32 | 2.36 | 2.41 | 2.46 | 2.53 | 2.60 | 2.69 | 2.81 | 2.97 | 3.21 | 3.61 | 4.71 | 4.55 | 3.52 | 3.10 | 2.85 | 2.67 | 2.53 | 2.42 | 2.32 | 2.24 | 2.17 | 2.10 | 2.04 | 1.99 | 1.94 | 1.89 | 1.84 |
| -0.90 | 2.15 | 2.17 | 2.20 | 2.24 | 2.27 | 2.32 | 2.36 | 2.42 | 2.49 | 2.57 | 2.68 | 2.82 | 3.03 | 3.40 | 4.46 | 4.03 | 3.25 | 2.91 | 2.70 | 2.55 | 2.43 | 2.33 | 2.25 | 2.18 | 2.12 | 2.06 | 2.01 | 1.96 | 1.92 | 1.88 | 1.84 |
| -1.00 | 2.08 | 2.11 | 2.13 | 2.16 | 2.20 | 2.24 | 2.28 | 2.33 | 2.39 | 2.47 | 2.57 | 2.70 | 2.89 | 3.22 | 4.27 | 3.64 | 3.03 | 2.75 | 2.57 | 2.44 | 2.34 | 2.26 | 2.19 | 2.12 | 2.07 | 2.02 | 1.98 | 1.93 | 1.90 | 1.86 | 1.83 |
| -1.10 | 2.02 | 2.05 | 2.07 | 2.10 | 2.13 | 2.17 | 2.21 | 2.25 | 2.31 | 2.38 | 2.47 | 2.59 | 2.76 | 3.07 | 4.15 | 3.35 | 2.86 | 2.62 | 2.47 | 2.35 | 2.26 | 2.19 | 2.13 | 2.07 | 2.03 | 1.98 | 1.94 | 1.91 | 1.87 | 1.84 | 1.81 |
| -1.20 | 1.97 | 2.00 | 2.02 | 2.04 | 2.07 | 2.11 | 2.14 | 2.19 | 2.24 | 2.30 | 2.39 | 2.50 | 2.66 | 2.95 | 4.11 | 3.11 | 2.71 | 2.51 | 2.37 | 2.27 | 2.20 | 2.13 | 2.08 | 2.03 | 1.98 | 1.95 | 1.91 | 1.88 | 1.85 | 1.82 | 1.79 |
| -1.30 | 1.93 | 1.95 | 1.97 | 2.00 | 2.02 | 2.05 | 2.09 | 2.13 | 2.18 | 2.24 | 2.31 | 2.41 | 2.57 | 2.84 | 4.18 | 2.92 | 2.58 | 2.41 | 2.29 | 2.20 | 2.13 | 2.08 | 2.03 | 1.98 | 1.95 | 1.91 | 1.88 | 1.85 | 1.82 | 1.80 | 1.78 |
| -1.40 | 1.89 | 1.91 | 1.93 | 1.95 | 1.98 | 2.00 | 2.04 | 2.08 | 2.12 | 2.18 | 2.25 | 2.34 | 2.48 | 2.75 | 4.64 | 2.76 | 2.48 | 2.32 | 2.22 | 2.14 | 2.08 | 2.03 | 1.98 | 1.94 | 1.91 | 1.88 | 1.85 | 1.82 | 1.80 | 1.78 | 1.76 |
| -1.50 | 1.85 | 1.87 | 1.89 | 1.91 | 1.94 | 1.96 | 1.99 | 2.03 | 2.07 | 2.12 | 2.19 | 2.28 | 2.41 | 2.67 | 4.31 | 2.63 | 2.39 | 2.25 | 2.16 | 2.09 | 2.03 | 1.98 | 1.94 | 1.91 | 1.88 | 1.85 | 1.82 | 1.80 | 1.78 | 1.75 | 1.74 |
| -1.60 | 1.82 | 1.84 | 1.86 | 1.88 | 1.90 | 1.92 | 1.95 | 1.99 | 2.03 | 2.07 | 2.14 | 2.22 | 2.35 | 2.59 | 3.51 | 2.52 | 2.31 | 2.18 | 2.10 | 2.04 | 1.99 | 1.94 | 1.91 | 1.87 | 1.84 | 1.82 | 1.79 | 1.77 | 1.75 | 1.73 | 1.72 |
| -1.70 | 1.79 | 1.81 | 1.82 | 1.84 | 1.86 | 1.89 | 1.91 | 1.95 | 1.98 | 2.03 | 2.09 | 2.17 | 2.29 | 2.53 | 3.14 | 2.42 | 2.23 | 2.13 | 2.05 | 1.99 | 1.94 | 1.91 | 1.87 | 1.84 | 1.81 | 1.79 | 1.77 | 1.75 | 1.73 | 1.71 | 1.70 |
| -1.80 | 1.76 | 1.78 | 1.79 | 1.81 | 1.83 | 1.86 | 1.88 | 1.91 | 1.95 | 1.99 | 2.05 | 2.12 | 2.24 | 2.47 | 2.89 | 2.34 | 2.17 | 2.07 | 2.00 | 1.95 | 1.91 | 1.87 | 1.84 | 1.81 | 1.79 | 1.77 | 1.75 | 1.73 | 1.71 | 1.69 | 1.68 |
| -1.90 | 1.74 | 1.75 | 1.77 | 1.79 | 1.80 | 1.83 | 1.85 | 1.88 | 1.91 | 1.95 | 2.01 | 2.08 | 2.19 | 2.42 | 2.71 | 2.26 | 2.12 | 2.03 | 1.96 | 1.91 | 1.87 | 1.84 | 1.81 | 1.78 | 1.76 | 1.74 | 1.72 | 1.71 | 1.69 | 1.67 | 1.66 |
| -2.00 | 1.72 | 1.73 | 1.74 | 1.76 | 1.78 | 1.80 | 1.82 | 1.85 | 1.88 | 1.92 | 1.97 | 2.04 | 2.15 | 2.37 | 2.57 | 2.20 | 2.06 | 1.98 | 1.92 | 1.88 | 1.84 | 1.81 | 1.78 | 1.76 | 1.74 | 1.72 | 1.70 | 1.69 | 1.67 | 1.66 | 1.64 |
| -2.10 | 1.69 | 1.71 | 1.72 | 1.74 | 1.75 | 1.77 | 1.79 | 1.82 | 1.85 | 1.89 | 1.94 | 2.00 | 2.11 | 2.33 | 2.46 | 2.14 | 2.02 | 1.94 | 1.89 | 1.85 | 1.81 | 1.78 | 1.76 | 1.74 | 1.72 | 1.70 | 1.68 | 1.67 | 1.65 | 1.64 | 1.63 |
| -2.20 | 1.67 | 1.69 | 1.70 | 1.71 | 1.73 | 1.75 | 1.77 | 1.79 | 1.82 | 1.86 | 1.91 | 1.97 | 2.07 | 2.30 | 2.36 | 2.09 | 1.98 | 1.91 | 1.86 | 1.82 | 1.78 | 1.76 | 1.73 | 1.71 | 1.69 | 1.68 | 1.66 | 1.65 | 1.64 | 1.62 | 1.61 |
| -2.30 | 1.65 | 1.67 | 1.68 | 1.69 | 1.71 | 1.73 | 1.75 | 1.77 | 1.80 | 1.83 | 1.88 | 1.94 | 2.04 | 2.26 | 2.28 | 2.04 | 1.94 | 1.87 | 1.83 | 1.79 | 1.76 | 1.73 | 1.71 | 1.69 | 1.67 | 1.66 | 1.64 | 1.63 | 1.62 | 1.61 | 1.60 |
| -2.40 | 1.64 | 1.65 | 1.66 | 1.67 | 1.69 | 1.71 | 1.73 | 1.75 | 1.77 | 1.81 | 1.85 | 1.91 | 2.01 | 2.23 | 2.21 | 2.00 | 1.90 | 1.84 | 1.80 | 1.76 | 1.74 | 1.71 | 1.69 | 1.67 | 1.66 | 1.64 | 1.63 | 1.61 | 1.60 | 1.59 | 1.58 |
| -2.50 | 1.62 | 1.63 | 1.64 | 1.66 | 1.67 | 1.69 | 1.70 | 1.73 | 1.75 | 1.78 | 1.83 | 1.88 | 1.98 | 2.20 | 2.14 | 1.96 | 1.87 | 1.81 | 1.77 | 1.74 | 1.71 | 1.69 | 1.67 | 1.65 | 1.64 | 1.62 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 |
| -2.60 | 1.60 | 1.61 | 1.63 | 1.64 | 1.65 | 1.67 | 1.69 | 1.71 | 1.73 | 1.76 | 1.80 | 1.86 | 1.95 | 2.18 | 2.09 | 1.92 | 1.84 | 1.79 | 1.75 | 1.72 | 1.69 | 1.67 | 1.65 | 1.64 | 1.62 | 1.61 | 1.60 | 1.58 | 1.57 | 1.56 | 1.56 |
| -2.70 | 1.59 | 1.60 | 1.61 | 1.62 | 1.64 | 1.65 | 1.67 | 1.69 | 1.71 | 1.74 | 1.78 | 1.83 | 1.92 | 2.16 | 2.04 | 1.89 | 1.81 | 1.76 | 1.73 | 1.70 | 1.67 | 1.65 | 1.64 | 1.62 | 1.61 | 1.59 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 |
| -2.80 | 1.57 | 1.58 | 1.59 | 1.61 | 1.62 | 1.63 | 1.65 | 1.67 | 1.69 | 1.72 | 1.76 | 1.81 | 1.90 | 2.14 | 1.99 | 1.85 | 1.79 | 1.74 | 1.70 | 1.68 | 1.66 | 1.64 | 1.62 | 1.60 | 1.59 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 |
| -2.90 | 1.56 | 1.57 | 1.58 | 1.59 | 1.60 | 1.62 | 1.63 | 1.65 | 1.68 | 1.70 | 1.74 | 1.79 | 1.88 | 2.13 | 1.95 | 1.83 | 1.76 | 1.72 | 1.69 | 1.66 | 1.64 | 1.62 | 1.60 | 1.59 | 1.58 | 1.56 | 1.55 | 1.54 | 1.54 | 1.53 | 1.52 |
| -3.00 | 1.55 | 1.56 | 1.57 | 1.58 | 1.59 | 1.60 | 1.62 | 1.64 | 1.66 | 1.69 | 1.72 | 1.77 | 1.85 | 2.12 | 1.92 | 1.80 | 1.74 | 1.70 | 1.67 | 1.64 | 1.62 | 1.60 | 1.59 | 1.58 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.52 | 1.51 |

🟦 $AREFF_{\overline{WH}|\overline{WH}^*} \geq 2$ 🟦 $1.2 \leq AREFF_{\overline{WH}|\overline{WH}^*} < 2$ 🟧 $1 \leq AREFF_{\overline{WH}|\overline{WH}^*} < 1.2$ ⬜ $AREFF_{\overline{WH}|\overline{WH}^*} < 1$

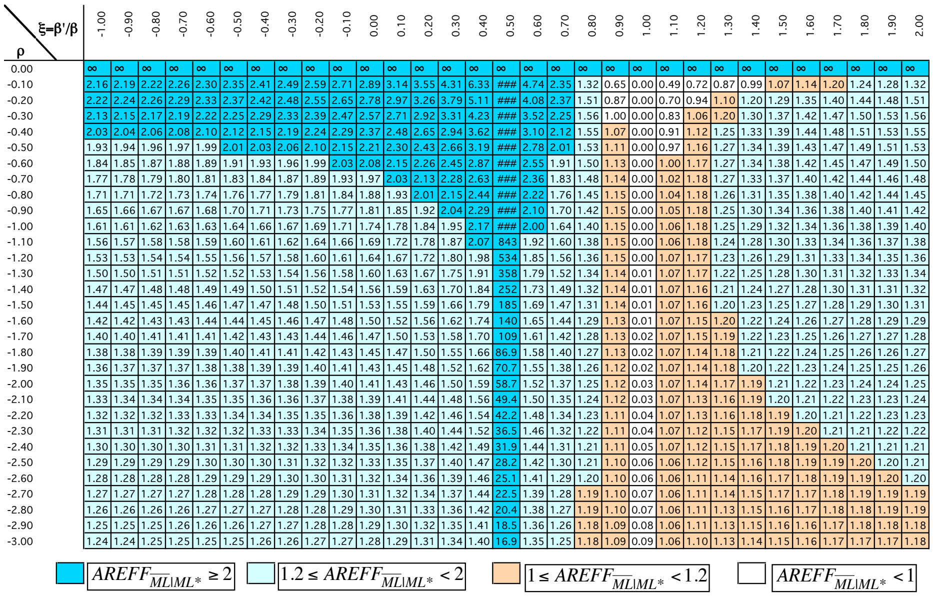
Figure 14: $AREFF_{\overline{WH}|\overline{WH}^*}$, in the $(\xi, \rho)$-plane.



| ρ \ ξ=β'/β | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 2.17 | 2.20 | 2.24 | 2.27 | 2.32 | 2.37 | 2.43 | 2.51 | 2.61 | 2.75 | 2.93 | 3.19 | 3.61 | 4.42 | 6.60 | ### | 4.68 | 2.37 | 1.35 | 0.69 | 0.11 | 0.45 | 0.69 | 0.85 | 0.97 | 1.06 | 1.13 | 1.18 | 1.23 | 1.27 | 1.31 |
| -0.20 | 2.25 | 2.28 | 2.30 | 2.34 | 2.37 | 2.42 | 2.47 | 2.54 | 2.62 | 2.73 | 2.88 | 3.09 | 3.43 | 4.06 | 5.74 | ### | 3.99 | 2.43 | 1.62 | 1.02 | 0.44 | 0.53 | 0.83 | 1.01 | 1.13 | 1.22 | 1.29 | 1.35 | 1.40 | 1.44 | 1.47 |
| -0.30 | 2.19 | 2.21 | 2.23 | 2.26 | 2.29 | 2.33 | 2.38 | 2.43 | 2.51 | 2.60 | 2.72 | 2.90 | 3.18 | 3.70 | 5.14 | 7.58 | 3.42 | 2.36 | 1.74 | 1.26 | 0.79 | 0.41 | 0.83 | 1.03 | 1.16 | 1.25 | 1.32 | 1.38 | 1.42 | 1.46 | 1.49 |
| -0.40 | 2.10 | 2.12 | 2.14 | 2.16 | 2.19 | 2.23 | 2.27 | 2.32 | 2.38 | 2.46 | 2.57 | 2.72 | 2.97 | 3.43 | 4.77 | 5.20 | 3.02 | 2.27 | 1.79 | 1.41 | 1.04 | 0.55 | 0.74 | 0.99 | 1.13 | 1.22 | 1.29 | 1.35 | 1.39 | 1.43 | 1.46 |
| -0.50 | 2.01 | 2.03 | 2.05 | 2.07 | 2.10 | 2.13 | 2.16 | 2.21 | 2.26 | 2.34 | 2.43 | 2.57 | 2.79 | 3.21 | 4.57 | 4.05 | 2.73 | 2.17 | 1.80 | 1.50 | 1.22 | 0.89 | 0.47 | 0.90 | 1.07 | 1.17 | 1.25 | 1.30 | 1.35 | 1.38 | 1.41 |
| -0.60 | 1.94 | 1.95 | 1.97 | 1.99 | 2.01 | 2.04 | 2.07 | 2.11 | 2.16 | 2.23 | 2.32 | 2.45 | 2.65 | 3.04 | 4.54 | 3.40 | 2.51 | 2.08 | 1.79 | 1.55 | 1.32 | 1.08 | 0.74 | 0.75 | 0.99 | 1.11 | 1.19 | 1.25 | 1.30 | 1.34 | 1.37 |
| -0.70 | 1.87 | 1.88 | 1.90 | 1.92 | 1.94 | 1.96 | 1.99 | 2.03 | 2.08 | 2.14 | 2.22 | 2.34 | 2.53 | 2.91 | 4.82 | 2.97 | 2.34 | 2.01 | 1.77 | 1.57 | 1.39 | 1.20 | 0.98 | 0.49 | 0.88 | 1.04 | 1.13 | 1.20 | 1.25 | 1.29 | 1.32 |
| -0.80 | 1.81 | 1.82 | 1.84 | 1.85 | 1.87 | 1.90 | 1.93 | 1.96 | 2.00 | 2.06 | 2.14 | 2.25 | 2.43 | 2.80 | 7.80 | 2.68 | 2.20 | 1.93 | 1.74 | 1.57 | 1.42 | 1.27 | 1.11 | 0.88 | 0.67 | 0.95 | 1.07 | 1.15 | 1.20 | 1.24 | 1.28 |
| -0.90 | 1.75 | 1.77 | 1.78 | 1.80 | 1.82 | 1.84 | 1.87 | 1.90 | 1.94 | 1.99 | 2.06 | 2.17 | 2.34 | 2.71 | 4.08 | 2.46 | 2.09 | 1.87 | 1.71 | 1.57 | 1.44 | 1.32 | 1.19 | 1.03 | 0.76 | 0.83 | 1.00 | 1.09 | 1.16 | 1.20 | 1.24 |
| -1.00 | 1.71 | 1.72 | 1.73 | 1.75 | 1.77 | 1.79 | 1.81 | 1.84 | 1.88 | 1.93 | 2.00 | 2.10 | 2.26 | 2.64 | 3.23 | 2.30 | 2.00 | 1.81 | 1.67 | 1.56 | 1.45 | 1.35 | 1.24 | 1.12 | 0.95 | 0.00 | 0.92 | 1.04 | 1.11 | 1.16 | 1.20 |
| -1.10 | 1.67 | 1.68 | 1.69 | 1.71 | 1.72 | 1.74 | 1.77 | 1.80 | 1.83 | 1.88 | 1.94 | 2.04 | 2.20 | 2.59 | 2.81 | 2.17 | 1.92 | 1.76 | 1.64 | 1.54 | 1.45 | 1.36 | 1.27 | 1.17 | 1.05 | 0.86 | 0.78 | 0.97 | 1.06 | 1.12 | 1.16 |
| -1.20 | 1.63 | 1.64 | 1.65 | 1.67 | 1.68 | 1.70 | 1.72 | 1.75 | 1.79 | 1.83 | 1.89 | 1.98 | 2.14 | 2.55 | 2.53 | 2.06 | 1.86 | 1.72 | 1.61 | 1.52 | 1.44 | 1.37 | 1.29 | 1.21 | 1.11 | 0.99 | 0.73 | 0.89 | 1.01 | 1.08 | 1.13 |
| -1.30 | 1.60 | 1.61 | 1.62 | 1.63 | 1.65 | 1.67 | 1.69 | 1.71 | 1.75 | 1.79 | 1.85 | 1.94 | 2.09 | 2.53 | 2.34 | 1.97 | 1.80 | 1.68 | 1.59 | 1.51 | 1.44 | 1.37 | 1.30 | 1.23 | 1.15 | 1.06 | 0.92 | 0.72 | 0.95 | 1.04 | 1.09 |
| -1.40 | 1.57 | 1.58 | 1.59 | 1.60 | 1.62 | 1.63 | 1.65 | 1.68 | 1.71 | 1.75 | 1.81 | 1.89 | 2.04 | 2.53 | 2.20 | 1.90 | 1.75 | 1.64 | 1.56 | 1.49 | 1.43 | 1.37 | 1.31 | 1.25 | 1.18 | 1.10 | 1.00 | 0.81 | 0.87 | 1.00 | 1.06 |
| -1.50 | 1.54 | 1.55 | 1.56 | 1.57 | 1.59 | 1.60 | 1.62 | 1.65 | 1.68 | 1.72 | 1.77 | 1.85 | 2.00 | 2.56 | 2.08 | 1.83 | 1.70 | 1.61 | 1.54 | 1.47 | 1.41 | 1.36 | 1.31 | 1.25 | 1.20 | 1.13 | 1.06 | 0.94 | 0.66 | 0.94 | 1.03 |
| -1.60 | 1.52 | 1.53 | 1.54 | 1.55 | 1.56 | 1.58 | 1.60 | 1.62 | 1.65 | 1.68 | 1.74 | 1.82 | 1.97 | 2.68 | 1.99 | 1.78 | 1.66 | 1.58 | 1.51 | 1.46 | 1.40 | 1.35 | 1.31 | 1.26 | 1.21 | 1.16 | 1.09 | 1.01 | 0.85 | 0.87 | 0.99 |
| -1.70 | 1.49 | 1.50 | 1.51 | 1.52 | 1.54 | 1.55 | 1.57 | 1.59 | 1.62 | 1.66 | 1.71 | 1.78 | 1.93 | 3.22 | 1.91 | 1.73 | 1.63 | 1.55 | 1.49 | 1.44 | 1.39 | 1.35 | 1.30 | 1.26 | 1.22 | 1.17 | 1.12 | 1.05 | 0.95 | 0.63 | 0.94 |
| -1.80 | 1.47 | 1.48 | 1.49 | 1.50 | 1.51 | 1.53 | 1.55 | 1.57 | 1.59 | 1.63 | 1.68 | 1.75 | 1.90 | 2.61 | 1.85 | 1.69 | 1.60 | 1.53 | 1.47 | 1.42 | 1.38 | 1.34 | 1.30 | 1.26 | 1.22 | 1.18 | 1.13 | 1.08 | 1.00 | 0.86 | 0.88 |
| -1.90 | 1.45 | 1.46 | 1.47 | 1.48 | 1.49 | 1.51 | 1.52 | 1.54 | 1.57 | 1.60 | 1.65 | 1.73 | 1.88 | 2.29 | 1.79 | 1.65 | 1.57 | 1.50 | 1.45 | 1.41 | 1.37 | 1.33 | 1.30 | 1.26 | 1.22 | 1.19 | 1.15 | 1.10 | 1.04 | 0.94 | 0.71 |
| -2.00 | 1.44 | 1.44 | 1.45 | 1.46 | 1.47 | 1.49 | 1.50 | 1.52 | 1.55 | 1.58 | 1.63 | 1.70 | 1.85 | 2.12 | 1.74 | 1.62 | 1.54 | 1.48 | 1.44 | 1.40 | 1.36 | 1.32 | 1.29 | 1.26 | 1.23 | 1.19 | 1.15 | 1.11 | 1.06 | 0.99 | 0.85 |
| -2.10 | 1.42 | 1.43 | 1.44 | 1.45 | 1.46 | 1.47 | 1.49 | 1.51 | 1.53 | 1.56 | 1.61 | 1.68 | 1.83 | 2.00 | 1.70 | 1.59 | 1.52 | 1.46 | 1.42 | 1.38 | 1.35 | 1.32 | 1.29 | 1.26 | 1.23 | 1.19 | 1.16 | 1.12 | 1.08 | 1.02 | 0.93 |
| -2.20 | 1.41 | 1.41 | 1.42 | 1.43 | 1.44 | 1.45 | 1.47 | 1.49 | 1.51 | 1.54 | 1.59 | 1.66 | 1.81 | 1.91 | 1.66 | 1.56 | 1.50 | 1.45 | 1.41 | 1.37 | 1.34 | 1.31 | 1.28 | 1.25 | 1.22 | 1.20 | 1.16 | 1.13 | 1.09 | 1.05 | 0.98 |
| -2.30 | 1.39 | 1.40 | 1.41 | 1.42 | 1.43 | 1.44 | 1.45 | 1.47 | 1.49 | 1.52 | 1.57 | 1.64 | 1.80 | 1.84 | 1.63 | 1.54 | 1.48 | 1.43 | 1.39 | 1.36 | 1.33 | 1.30 | 1.27 | 1.25 | 1.22 | 1.20 | 1.17 | 1.14 | 1.10 | 1.06 | 1.01 |
| -2.40 | 1.38 | 1.39 | 1.39 | 1.40 | 1.41 | 1.42 | 1.44 | 1.46 | 1.48 | 1.51 | 1.55 | 1.62 | 1.78 | 1.78 | 1.60 | 1.51 | 1.46 | 1.41 | 1.38 | 1.35 | 1.32 | 1.29 | 1.27 | 1.24 | 1.22 | 1.19 | 1.17 | 1.14 | 1.11 | 1.07 | 1.03 |
| -2.50 | 1.37 | 1.37 | 1.38 | 1.39 | 1.40 | 1.41 | 1.42 | 1.44 | 1.46 | 1.49 | 1.53 | 1.60 | 1.77 | 1.73 | 1.57 | 1.49 | 1.44 | 1.40 | 1.37 | 1.34 | 1.31 | 1.29 | 1.26 | 1.24 | 1.22 | 1.19 | 1.17 | 1.14 | 1.12 | 1.08 | 1.04 |
| -2.60 | 1.35 | 1.36 | 1.37 | 1.38 | 1.39 | 1.40 | 1.41 | 1.43 | 1.45 | 1.48 | 1.52 | 1.58 | 1.76 | 1.69 | 1.54 | 1.47 | 1.42 | 1.39 | 1.35 | 1.33 | 1.30 | 1.28 | 1.26 | 1.24 | 1.21 | 1.19 | 1.17 | 1.15 | 1.12 | 1.09 | 1.06 |
| -2.70 | 1.34 | 1.35 | 1.36 | 1.37 | 1.37 | 1.39 | 1.40 | 1.41 | 1.43 | 1.46 | 1.50 | 1.57 | 1.76 | 1.65 | 1.52 | 1.45 | 1.41 | 1.37 | 1.34 | 1.32 | 1.29 | 1.27 | 1.25 | 1.23 | 1.21 | 1.19 | 1.17 | 1.15 | 1.12 | 1.10 | 1.07 |
| -2.80 | 1.33 | 1.34 | 1.35 | 1.35 | 1.36 | 1.37 | 1.39 | 1.40 | 1.42 | 1.45 | 1.49 | 1.55 | 1.75 | 1.62 | 1.50 | 1.44 | 1.39 | 1.36 | 1.33 | 1.31 | 1.29 | 1.27 | 1.25 | 1.23 | 1.21 | 1.19 | 1.17 | 1.15 | 1.13 | 1.10 | 1.07 |
| -2.90 | 1.32 | 1.33 | 1.34 | 1.34 | 1.35 | 1.36 | 1.38 | 1.39 | 1.41 | 1.44 | 1.47 | 1.54 | 1.75 | 1.59 | 1.48 | 1.42 | 1.38 | 1.35 | 1.32 | 1.30 | 1.28 | 1.26 | 1.24 | 1.22 | 1.20 | 1.19 | 1.17 | 1.15 | 1.13 | 1.11 | 1.08 |
| -3.00 | 1.32 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.37 | 1.38 | 1.40 | 1.42 | 1.46 | 1.53 | 1.75 | 1.56 | 1.46 | 1.41 | 1.37 | 1.34 | 1.31 | 1.29 | 1.27 | 1.25 | 1.23 | 1.22 | 1.20 | 1.18 | 1.17 | 1.15 | 1.13 | 1.11 | 1.08 |

🟦 $AREFF_{\overline{CH}|CH^*} \geq 2$ 🟦 $1.2 \leq AREFF_{\overline{CH}|CH^*} < 2$ 🟧 $1 \leq AREFF_{\overline{CH}|CH^*} < 1.2$ ⬜ $AREFF_{\overline{CH}|CH^*} < 1$

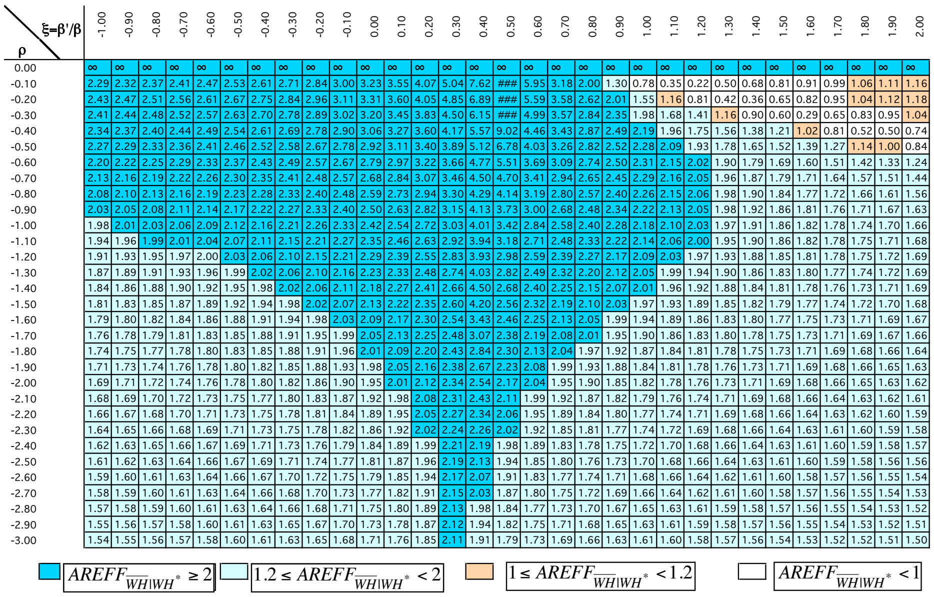
Figure 15: $AREFF_{\overline{CH}|CH^*}$, in the $(\xi, \rho)$-plane.



| ρ \ ξ=β'/β | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 2.16 | 2.19 | 2.22 | 2.26 | 2.30 | 2.35 | 2.41 | 2.49 | 2.59 | 2.71 | 2.89 | 3.14 | 3.55 | 4.31 | 6.33 | ### | 4.74 | 2.35 | 1.32 | 0.65 | 0.00 | 0.49 | 0.72 | 0.87 | 0.99 | 1.07 | 1.14 | 1.20 | 1.24 | 1.28 | 1.32 |
| -0.20 | 2.22 | 2.24 | 2.26 | 2.29 | 2.33 | 2.37 | 2.42 | 2.48 | 2.55 | 2.65 | 2.78 | 2.97 | 3.26 | 3.79 | 5.11 | ### | 4.08 | 2.37 | 1.51 | 0.87 | 0.00 | 0.70 | 0.94 | 1.10 | 1.20 | 1.29 | 1.35 | 1.40 | 1.44 | 1.48 | 1.51 |
| -0.30 | 2.13 | 2.15 | 2.17 | 2.19 | 2.22 | 2.25 | 2.29 | 2.33 | 2.39 | 2.47 | 2.57 | 2.71 | 2.92 | 3.31 | 4.23 | ### | 3.52 | 2.25 | 1.56 | 1.00 | 0.00 | 0.83 | 1.06 | 1.20 | 1.30 | 1.37 | 1.42 | 1.47 | 1.50 | 1.53 | 1.56 |
| -0.40 | 2.03 | 2.04 | 2.06 | 2.08 | 2.10 | 2.12 | 2.15 | 2.19 | 2.24 | 2.29 | 2.37 | 2.48 | 2.65 | 2.94 | 3.62 | ### | 3.10 | 2.12 | 1.55 | 1.07 | 0.00 | 0.91 | 1.12 | 1.25 | 1.33 | 1.39 | 1.44 | 1.48 | 1.51 | 1.53 | 1.55 |
| -0.50 | 1.93 | 1.94 | 1.96 | 1.97 | 1.99 | 2.01 | 2.03 | 2.06 | 2.10 | 2.15 | 2.21 | 2.30 | 2.43 | 2.66 | 3.19 | ### | 2.78 | 2.01 | 1.53 | 1.11 | 0.00 | 0.97 | 1.16 | 1.27 | 1.34 | 1.39 | 1.43 | 1.47 | 1.49 | 1.51 | 1.53 |
| -0.60 | 1.84 | 1.85 | 1.87 | 1.88 | 1.89 | 1.91 | 1.93 | 1.96 | 1.99 | 2.03 | 2.08 | 2.15 | 2.26 | 2.45 | 2.87 | ### | 2.55 | 1.91 | 1.50 | 1.13 | 0.00 | 1.00 | 1.17 | 1.27 | 1.34 | 1.38 | 1.42 | 1.45 | 1.47 | 1.49 | 1.50 |
| -0.70 | 1.77 | 1.78 | 1.79 | 1.80 | 1.81 | 1.83 | 1.84 | 1.87 | 1.89 | 1.93 | 1.97 | 2.03 | 2.13 | 2.28 | 2.63 | ### | 2.36 | 1.83 | 1.48 | 1.14 | 0.00 | 1.02 | 1.18 | 1.27 | 1.33 | 1.37 | 1.40 | 1.42 | 1.44 | 1.46 | 1.48 |
| -0.80 | 1.71 | 1.71 | 1.72 | 1.73 | 1.74 | 1.76 | 1.77 | 1.79 | 1.81 | 1.84 | 1.88 | 1.93 | 2.01 | 2.15 | 2.44 | ### | 2.22 | 1.76 | 1.45 | 1.15 | 0.00 | 1.04 | 1.18 | 1.26 | 1.31 | 1.35 | 1.38 | 1.40 | 1.42 | 1.44 | 1.45 |
| -0.90 | 1.65 | 1.66 | 1.67 | 1.67 | 1.68 | 1.70 | 1.71 | 1.73 | 1.75 | 1.77 | 1.81 | 1.85 | 1.92 | 2.04 | 2.29 | ### | 2.10 | 1.70 | 1.42 | 1.15 | 0.00 | 1.05 | 1.18 | 1.25 | 1.30 | 1.34 | 1.36 | 1.38 | 1.40 | 1.41 | 1.42 |
| -1.00 | 1.61 | 1.61 | 1.62 | 1.63 | 1.63 | 1.64 | 1.66 | 1.67 | 1.69 | 1.71 | 1.74 | 1.78 | 1.84 | 1.95 | 2.17 | ### | 2.00 | 1.64 | 1.40 | 1.15 | 0.00 | 1.06 | 1.18 | 1.25 | 1.29 | 1.32 | 1.34 | 1.36 | 1.38 | 1.39 | 1.40 |
| -1.10 | 1.56 | 1.57 | 1.58 | 1.58 | 1.59 | 1.60 | 1.61 | 1.62 | 1.64 | 1.66 | 1.69 | 1.72 | 1.78 | 1.87 | 2.07 | 843 | 1.92 | 1.60 | 1.38 | 1.15 | 0.00 | 1.06 | 1.18 | 1.24 | 1.28 | 1.30 | 1.33 | 1.34 | 1.36 | 1.37 | 1.38 |
| -1.20 | 1.53 | 1.53 | 1.54 | 1.54 | 1.55 | 1.56 | 1.57 | 1.58 | 1.60 | 1.61 | 1.64 | 1.67 | 1.72 | 1.80 | 1.98 | 534 | 1.85 | 1.56 | 1.36 | 1.15 | 0.00 | 1.07 | 1.17 | 1.23 | 1.26 | 1.29 | 1.31 | 1.33 | 1.34 | 1.35 | 1.36 |
| -1.30 | 1.50 | 1.50 | 1.51 | 1.51 | 1.52 | 1.52 | 1.53 | 1.54 | 1.56 | 1.58 | 1.60 | 1.63 | 1.67 | 1.75 | 1.91 | 358 | 1.79 | 1.52 | 1.34 | 1.14 | 0.01 | 1.07 | 1.17 | 1.22 | 1.25 | 1.28 | 1.30 | 1.31 | 1.32 | 1.33 | 1.34 |
| -1.40 | 1.47 | 1.47 | 1.48 | 1.48 | 1.49 | 1.49 | 1.50 | 1.51 | 1.52 | 1.54 | 1.56 | 1.59 | 1.63 | 1.70 | 1.84 | 252 | 1.73 | 1.49 | 1.32 | 1.14 | 0.01 | 1.07 | 1.16 | 1.21 | 1.24 | 1.27 | 1.28 | 1.30 | 1.31 | 1.31 | 1.32 |
| -1.50 | 1.44 | 1.45 | 1.45 | 1.45 | 1.46 | 1.47 | 1.47 | 1.48 | 1.50 | 1.51 | 1.53 | 1.55 | 1.59 | 1.66 | 1.79 | 185 | 1.69 | 1.47 | 1.31 | 1.14 | 0.01 | 1.07 | 1.16 | 1.20 | 1.23 | 1.25 | 1.27 | 1.28 | 1.29 | 1.30 | 1.31 |
| -1.60 | 1.42 | 1.42 | 1.43 | 1.43 | 1.44 | 1.44 | 1.45 | 1.46 | 1.47 | 1.48 | 1.50 | 1.52 | 1.56 | 1.62 | 1.74 | 140 | 1.65 | 1.44 | 1.29 | 1.13 | 0.01 | 1.07 | 1.15 | 1.20 | 1.22 | 1.24 | 1.26 | 1.27 | 1.28 | 1.29 | 1.29 |
| -1.70 | 1.40 | 1.40 | 1.41 | 1.41 | 1.41 | 1.42 | 1.43 | 1.43 | 1.44 | 1.46 | 1.47 | 1.50 | 1.53 | 1.58 | 1.70 | 109 | 1.61 | 1.42 | 1.28 | 1.13 | 0.02 | 1.07 | 1.15 | 1.19 | 1.22 | 1.23 | 1.25 | 1.26 | 1.27 | 1.27 | 1.28 |
| -1.80 | 1.38 | 1.38 | 1.39 | 1.39 | 1.39 | 1.40 | 1.41 | 1.41 | 1.42 | 1.43 | 1.45 | 1.47 | 1.50 | 1.55 | 1.66 | 86.9 | 1.58 | 1.40 | 1.27 | 1.13 | 0.02 | 1.07 | 1.14 | 1.18 | 1.21 | 1.22 | 1.24 | 1.25 | 1.26 | 1.26 | 1.27 |
| -1.90 | 1.36 | 1.37 | 1.37 | 1.37 | 1.38 | 1.38 | 1.39 | 1.39 | 1.40 | 1.41 | 1.43 | 1.45 | 1.48 | 1.52 | 1.62 | 70.7 | 1.55 | 1.38 | 1.26 | 1.12 | 0.02 | 1.07 | 1.14 | 1.18 | 1.20 | 1.22 | 1.23 | 1.24 | 1.25 | 1.25 | 1.26 |
| -2.00 | 1.35 | 1.35 | 1.35 | 1.36 | 1.36 | 1.37 | 1.37 | 1.38 | 1.39 | 1.40 | 1.41 | 1.43 | 1.46 | 1.50 | 1.59 | 58.7 | 1.52 | 1.37 | 1.25 | 1.12 | 0.03 | 1.07 | 1.14 | 1.17 | 1.19 | 1.21 | 1.22 | 1.23 | 1.24 | 1.24 | 1.25 |
| -2.10 | 1.33 | 1.34 | 1.34 | 1.34 | 1.35 | 1.35 | 1.36 | 1.36 | 1.37 | 1.38 | 1.39 | 1.41 | 1.44 | 1.48 | 1.56 | 49.4 | 1.50 | 1.35 | 1.24 | 1.12 | 0.03 | 1.07 | 1.13 | 1.16 | 1.19 | 1.20 | 1.21 | 1.22 | 1.23 | 1.23 | 1.24 |
| -2.20 | 1.32 | 1.32 | 1.32 | 1.33 | 1.33 | 1.34 | 1.34 | 1.35 | 1.35 | 1.36 | 1.38 | 1.39 | 1.42 | 1.46 | 1.54 | 42.2 | 1.48 | 1.34 | 1.23 | 1.11 | 0.04 | 1.07 | 1.13 | 1.16 | 1.18 | 1.19 | 1.20 | 1.21 | 1.22 | 1.23 | 1.23 |
| -2.30 | 1.31 | 1.31 | 1.31 | 1.32 | 1.32 | 1.32 | 1.33 | 1.33 | 1.34 | 1.35 | 1.36 | 1.38 | 1.40 | 1.44 | 1.52 | 36.5 | 1.46 | 1.32 | 1.22 | 1.11 | 0.04 | 1.07 | 1.12 | 1.15 | 1.17 | 1.19 | 1.20 | 1.21 | 1.21 | 1.22 | 1.22 |
| -2.40 | 1.30 | 1.30 | 1.30 | 1.30 | 1.31 | 1.31 | 1.32 | 1.32 | 1.33 | 1.34 | 1.35 | 1.36 | 1.38 | 1.42 | 1.49 | 31.9 | 1.44 | 1.31 | 1.21 | 1.11 | 0.05 | 1.07 | 1.12 | 1.15 | 1.17 | 1.18 | 1.19 | 1.20 | 1.21 | 1.21 | 1.21 |
| -2.50 | 1.29 | 1.29 | 1.29 | 1.29 | 1.30 | 1.30 | 1.30 | 1.31 | 1.32 | 1.32 | 1.33 | 1.35 | 1.37 | 1.40 | 1.47 | 28.2 | 1.42 | 1.30 | 1.21 | 1.10 | 0.06 | 1.06 | 1.12 | 1.15 | 1.16 | 1.18 | 1.19 | 1.19 | 1.20 | 1.20 | 1.21 |
| -2.60 | 1.28 | 1.28 | 1.28 | 1.28 | 1.29 | 1.29 | 1.29 | 1.30 | 1.30 | 1.31 | 1.32 | 1.34 | 1.36 | 1.39 | 1.46 | 25.1 | 1.41 | 1.29 | 1.20 | 1.10 | 0.06 | 1.06 | 1.11 | 1.14 | 1.16 | 1.17 | 1.18 | 1.19 | 1.19 | 1.20 | 1.20 |
| -2.70 | 1.27 | 1.27 | 1.27 | 1.27 | 1.28 | 1.28 | 1.28 | 1.29 | 1.29 | 1.30 | 1.31 | 1.32 | 1.34 | 1.37 | 1.44 | 22.5 | 1.39 | 1.28 | 1.19 | 1.10 | 0.07 | 1.06 | 1.11 | 1.14 | 1.15 | 1.17 | 1.17 | 1.18 | 1.19 | 1.19 | 1.19 |
| -2.80 | 1.26 | 1.26 | 1.26 | 1.26 | 1.27 | 1.27 | 1.27 | 1.28 | 1.28 | 1.29 | 1.30 | 1.31 | 1.33 | 1.36 | 1.42 | 20.4 | 1.38 | 1.27 | 1.19 | 1.10 | 0.07 | 1.06 | 1.11 | 1.13 | 1.15 | 1.16 | 1.17 | 1.18 | 1.18 | 1.19 | 1.19 |
| -2.90 | 1.25 | 1.25 | 1.25 | 1.26 | 1.26 | 1.26 | 1.27 | 1.27 | 1.28 | 1.28 | 1.29 | 1.30 | 1.32 | 1.35 | 1.41 | 18.5 | 1.36 | 1.26 | 1.18 | 1.09 | 0.08 | 1.06 | 1.11 | 1.13 | 1.15 | 1.16 | 1.16 | 1.17 | 1.18 | 1.18 | 1.18 |
| -3.00 | 1.24 | 1.24 | 1.25 | 1.25 | 1.25 | 1.25 | 1.26 | 1.26 | 1.27 | 1.27 | 1.28 | 1.29 | 1.31 | 1.34 | 1.40 | 16.9 | 1.35 | 1.25 | 1.18 | 1.09 | 0.09 | 1.06 | 1.10 | 1.13 | 1.14 | 1.15 | 1.16 | 1.17 | 1.17 | 1.17 | 1.18 |

🟦 $AREFF_{\overline{ML}|ML^*} \geq 2$
▨ $1.2 \leq AREFF_{\overline{ML}|ML^*} < 2$
🟧 $1 \leq AREFF_{\overline{ML}|ML^*} < 1.2$
☐ $AREFF_{\overline{ML}|ML^*} < 1$

Figure 16: $AREFF_{\overline{ML}|ML^*}$, in the $(\xi, \rho)$-plane.



| ρ \ ξ=β'/β | -1.00 | -0.90 | -0.80 | -0.70 | -0.60 | -0.50 | -0.40 | -0.30 | -0.20 | -0.10 | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.80 | 0.90 | 1.00 | 1.10 | 1.20 | 1.30 | 1.40 | 1.50 | 1.60 | 1.70 | 1.80 | 1.90 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.00 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| -0.10 | 2.29 | 2.32 | 2.37 | 2.41 | 2.47 | 2.53 | 2.61 | 2.71 | 2.84 | 3.00 | 3.23 | 3.55 | 4.07 | 5.04 | 7.62 | ### | 5.95 | 3.18 | 2.00 | 1.30 | 0.78 | 0.35 | 0.22 | 0.50 | 0.68 | 0.81 | 0.91 | 0.99 | 1.06 | 1.11 | 1.16 |
| -0.20 | 2.43 | 2.47 | 2.51 | 2.56 | 2.61 | 2.67 | 2.75 | 2.84 | 2.96 | 3.11 | 3.31 | 3.60 | 4.05 | 4.85 | 6.89 | ### | 5.59 | 3.58 | 2.62 | 2.01 | 1.55 | 1.16 | 0.81 | 0.42 | 0.36 | 0.65 | 0.82 | 0.95 | 1.04 | 1.12 | 1.18 |
| -0.30 | 2.41 | 2.44 | 2.48 | 2.52 | 2.57 | 2.63 | 2.70 | 2.78 | 2.89 | 3.02 | 3.20 | 3.45 | 3.83 | 4.50 | 6.15 | ### | 4.99 | 3.57 | 2.84 | 2.35 | 1.98 | 1.68 | 1.41 | 1.16 | 0.90 | 0.60 | 0.29 | 0.65 | 0.83 | 0.95 | 1.04 |
| -0.40 | 2.34 | 2.37 | 2.40 | 2.44 | 2.49 | 2.54 | 2.61 | 2.69 | 2.78 | 2.90 | 3.06 | 3.27 | 3.60 | 4.17 | 5.57 | 9.02 | 4.46 | 3.43 | 2.87 | 2.49 | 2.19 | 1.96 | 1.75 | 1.56 | 1.38 | 1.21 | 1.02 | 0.81 | 0.52 | 0.50 | 0.74 |
| -0.50 | 2.27 | 2.29 | 2.33 | 2.36 | 2.41 | 2.46 | 2.52 | 2.58 | 2.67 | 2.78 | 2.92 | 3.11 | 3.40 | 3.89 | 5.12 | 6.78 | 4.03 | 3.26 | 2.82 | 2.52 | 2.28 | 2.09 | 1.93 | 1.78 | 1.65 | 1.52 | 1.39 | 1.27 | 1.14 | 1.00 | 0.84 |
| -0.60 | 2.20 | 2.22 | 2.25 | 2.29 | 2.33 | 2.37 | 2.43 | 2.49 | 2.57 | 2.67 | 2.79 | 2.97 | 3.22 | 3.66 | 4.77 | 5.51 | 3.69 | 3.09 | 2.74 | 2.50 | 2.31 | 2.15 | 2.02 | 1.90 | 1.79 | 1.69 | 1.60 | 1.51 | 1.42 | 1.33 | 1.24 |
| -0.70 | 2.13 | 2.16 | 2.19 | 2.22 | 2.26 | 2.30 | 2.35 | 2.41 | 2.48 | 2.57 | 2.68 | 2.84 | 3.07 | 3.46 | 4.50 | 4.70 | 3.41 | 2.94 | 2.65 | 2.45 | 2.29 | 2.16 | 2.05 | 1.96 | 1.87 | 1.79 | 1.71 | 1.64 | 1.57 | 1.51 | 1.44 |
| -0.80 | 2.08 | 2.10 | 2.13 | 2.16 | 2.19 | 2.23 | 2.28 | 2.33 | 2.40 | 2.48 | 2.59 | 2.73 | 2.94 | 3.30 | 4.29 | 4.14 | 3.19 | 2.80 | 2.57 | 2.40 | 2.26 | 2.15 | 2.06 | 1.98 | 1.90 | 1.84 | 1.77 | 1.72 | 1.66 | 1.61 | 1.56 |
| -0.90 | 2.03 | 2.05 | 2.08 | 2.11 | 2.14 | 2.17 | 2.22 | 2.27 | 2.33 | 2.40 | 2.50 | 2.63 | 2.82 | 3.15 | 4.13 | 3.73 | 3.00 | 2.68 | 2.48 | 2.34 | 2.22 | 2.13 | 2.05 | 1.98 | 1.92 | 1.86 | 1.81 | 1.76 | 1.71 | 1.67 | 1.63 |
| -1.00 | 1.98 | 2.01 | 2.03 | 2.06 | 2.09 | 2.12 | 2.16 | 2.21 | 2.26 | 2.33 | 2.42 | 2.54 | 2.72 | 3.03 | 4.01 | 3.42 | 2.84 | 2.58 | 2.40 | 2.28 | 2.18 | 2.10 | 2.03 | 1.97 | 1.91 | 1.86 | 1.82 | 1.78 | 1.74 | 1.70 | 1.66 |
| -1.10 | 1.94 | 1.96 | 1.99 | 2.01 | 2.04 | 2.07 | 2.11 | 2.15 | 2.21 | 2.27 | 2.35 | 2.46 | 2.63 | 2.92 | 3.94 | 3.18 | 2.71 | 2.48 | 2.33 | 2.22 | 2.14 | 2.06 | 2.00 | 1.95 | 1.90 | 1.86 | 1.82 | 1.78 | 1.75 | 1.71 | 1.68 |
| -1.20 | 1.91 | 1.93 | 1.95 | 1.97 | 2.00 | 2.03 | 2.06 | 2.10 | 2.15 | 2.21 | 2.29 | 2.39 | 2.55 | 2.83 | 3.93 | 2.98 | 2.59 | 2.39 | 2.27 | 2.17 | 2.09 | 2.03 | 1.97 | 1.93 | 1.88 | 1.85 | 1.81 | 1.78 | 1.75 | 1.72 | 1.69 |
| -1.30 | 1.87 | 1.89 | 1.91 | 1.93 | 1.96 | 1.99 | 2.02 | 2.06 | 2.10 | 2.16 | 2.23 | 2.33 | 2.48 | 2.74 | 4.03 | 2.82 | 2.49 | 2.32 | 2.20 | 2.12 | 2.05 | 1.99 | 1.94 | 1.90 | 1.86 | 1.83 | 1.80 | 1.77 | 1.74 | 1.72 | 1.69 |
| -1.40 | 1.84 | 1.86 | 1.88 | 1.90 | 1.92 | 1.95 | 1.98 | 2.02 | 2.06 | 2.11 | 2.18 | 2.27 | 2.41 | 2.66 | 4.50 | 2.68 | 2.40 | 2.25 | 2.15 | 2.07 | 2.01 | 1.96 | 1.92 | 1.88 | 1.84 | 1.81 | 1.78 | 1.76 | 1.73 | 1.71 | 1.69 |
| -1.50 | 1.81 | 1.83 | 1.85 | 1.87 | 1.89 | 1.92 | 1.94 | 1.98 | 2.02 | 2.07 | 2.13 | 2.22 | 2.35 | 2.60 | 4.20 | 2.56 | 2.32 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.89 | 1.85 | 1.82 | 1.79 | 1.77 | 1.74 | 1.72 | 1.70 | 1.68 |
| -1.60 | 1.79 | 1.80 | 1.82 | 1.84 | 1.86 | 1.88 | 1.91 | 1.94 | 1.98 | 2.03 | 2.09 | 2.17 | 2.30 | 2.54 | 3.43 | 2.46 | 2.25 | 2.13 | 2.05 | 1.99 | 1.94 | 1.89 | 1.86 | 1.83 | 1.80 | 1.77 | 1.75 | 1.73 | 1.71 | 1.69 | 1.67 |
| -1.70 | 1.76 | 1.78 | 1.79 | 1.81 | 1.83 | 1.85 | 1.88 | 1.91 | 1.95 | 1.99 | 2.05 | 2.13 | 2.25 | 2.48 | 3.07 | 2.38 | 2.19 | 2.08 | 2.01 | 1.95 | 1.90 | 1.86 | 1.83 | 1.80 | 1.78 | 1.75 | 1.73 | 1.71 | 1.69 | 1.67 | 1.66 |
| -1.80 | 1.74 | 1.75 | 1.77 | 1.78 | 1.80 | 1.83 | 1.85 | 1.88 | 1.91 | 1.96 | 2.01 | 2.09 | 2.20 | 2.43 | 2.84 | 2.30 | 2.13 | 2.04 | 1.97 | 1.92 | 1.87 | 1.84 | 1.81 | 1.78 | 1.75 | 1.73 | 1.71 | 1.69 | 1.68 | 1.66 | 1.64 |
| -1.90 | 1.71 | 1.73 | 1.74 | 1.76 | 1.78 | 1.80 | 1.82 | 1.85 | 1.88 | 1.93 | 1.98 | 2.05 | 2.16 | 2.38 | 2.67 | 2.23 | 2.08 | 1.99 | 1.93 | 1.88 | 1.84 | 1.81 | 1.78 | 1.76 | 1.73 | 1.71 | 1.69 | 1.68 | 1.66 | 1.65 | 1.63 |
| -2.00 | 1.69 | 1.71 | 1.72 | 1.74 | 1.76 | 1.78 | 1.80 | 1.82 | 1.86 | 1.90 | 1.95 | 2.01 | 2.12 | 2.34 | 2.54 | 2.17 | 2.04 | 1.95 | 1.90 | 1.85 | 1.82 | 1.78 | 1.76 | 1.73 | 1.71 | 1.69 | 1.68 | 1.66 | 1.65 | 1.63 | 1.62 |
| -2.10 | 1.68 | 1.69 | 1.70 | 1.72 | 1.73 | 1.75 | 1.77 | 1.80 | 1.83 | 1.87 | 1.92 | 1.98 | 2.08 | 2.31 | 2.43 | 2.11 | 1.99 | 1.92 | 1.87 | 1.82 | 1.79 | 1.76 | 1.74 | 1.71 | 1.69 | 1.68 | 1.66 | 1.64 | 1.63 | 1.62 | 1.61 |
| -2.20 | 1.66 | 1.67 | 1.68 | 1.70 | 1.71 | 1.73 | 1.75 | 1.78 | 1.81 | 1.84 | 1.89 | 1.95 | 2.05 | 2.27 | 2.34 | 2.06 | 1.95 | 1.89 | 1.84 | 1.80 | 1.77 | 1.74 | 1.71 | 1.69 | 1.68 | 1.66 | 1.64 | 1.63 | 1.62 | 1.60 | 1.59 |
| -2.30 | 1.64 | 1.65 | 1.66 | 1.68 | 1.69 | 1.71 | 1.73 | 1.75 | 1.78 | 1.82 | 1.86 | 1.92 | 2.02 | 2.24 | 2.26 | 2.02 | 1.92 | 1.85 | 1.81 | 1.77 | 1.74 | 1.72 | 1.69 | 1.68 | 1.66 | 1.64 | 1.63 | 1.61 | 1.60 | 1.59 | 1.58 |
| -2.40 | 1.62 | 1.63 | 1.65 | 1.66 | 1.67 | 1.69 | 1.71 | 1.73 | 1.76 | 1.79 | 1.84 | 1.89 | 1.99 | 2.21 | 2.19 | 1.98 | 1.89 | 1.83 | 1.78 | 1.75 | 1.72 | 1.70 | 1.68 | 1.66 | 1.64 | 1.63 | 1.61 | 1.60 | 1.59 | 1.58 | 1.57 |
| -2.50 | 1.61 | 1.62 | 1.63 | 1.64 | 1.66 | 1.67 | 1.69 | 1.71 | 1.74 | 1.77 | 1.81 | 1.87 | 1.96 | 2.19 | 2.13 | 1.94 | 1.85 | 1.80 | 1.76 | 1.73 | 1.70 | 1.68 | 1.66 | 1.64 | 1.63 | 1.61 | 1.60 | 1.59 | 1.58 | 1.56 | 1.56 |
| -2.60 | 1.59 | 1.60 | 1.61 | 1.63 | 1.64 | 1.66 | 1.67 | 1.70 | 1.72 | 1.75 | 1.79 | 1.85 | 1.94 | 2.17 | 2.07 | 1.91 | 1.83 | 1.77 | 1.74 | 1.71 | 1.68 | 1.66 | 1.64 | 1.62 | 1.61 | 1.60 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 |
| -2.70 | 1.58 | 1.59 | 1.60 | 1.61 | 1.63 | 1.64 | 1.66 | 1.68 | 1.70 | 1.73 | 1.77 | 1.82 | 1.91 | 2.15 | 2.03 | 1.87 | 1.80 | 1.75 | 1.72 | 1.69 | 1.66 | 1.64 | 1.62 | 1.61 | 1.60 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 |
| -2.80 | 1.57 | 1.58 | 1.59 | 1.60 | 1.61 | 1.63 | 1.64 | 1.66 | 1.68 | 1.71 | 1.75 | 1.80 | 1.89 | 2.13 | 1.98 | 1.84 | 1.77 | 1.73 | 1.70 | 1.67 | 1.65 | 1.63 | 1.61 | 1.59 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 |
| -2.90 | 1.55 | 1.56 | 1.57 | 1.58 | 1.60 | 1.61 | 1.63 | 1.65 | 1.67 | 1.70 | 1.73 | 1.78 | 1.87 | 2.12 | 1.94 | 1.82 | 1.75 | 1.71 | 1.68 | 1.65 | 1.63 | 1.61 | 1.59 | 1.58 | 1.57 | 1.56 | 1.55 | 1.54 | 1.53 | 1.52 | 1.51 |
| -3.00 | 1.54 | 1.55 | 1.56 | 1.57 | 1.58 | 1.60 | 1.61 | 1.63 | 1.65 | 1.68 | 1.71 | 1.76 | 1.85 | 2.11 | 1.91 | 1.79 | 1.73 | 1.69 | 1.66 | 1.63 | 1.61 | 1.60 | 1.58 | 1.57 | 1.56 | 1.54 | 1.53 | 1.52 | 1.52 | 1.51 | 1.50 |

🟦 $AREFF_{\overline{WH}|WH^*} \geq 2$
▨ $1.2 \leq AREFF_{\overline{WH}|WH^*} < 2$
🟧 $1 \leq AREFF_{\overline{WH}|WH^*} < 1.2$
☐ $AREFF_{\overline{WH}|WH^*} < 1$

Figure 17: $AREFF_{\overline{WH}|WH^*}$, in the $(\xi, \rho)$-plane.

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